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Development of an aircraft performance model for the prediction of trip fuel and trip time for a generic twin engine jet transport aircraft

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Abstract

This report gives an overview of methods for aircraft performance calculations. After explaining the necessary background and the International Standard Atmosphere, it deals with a complete mission of a generic twin engine jet transport aircraft, including the required reserves of a diversion. Every part of the mission is considered. This includes climb, cruise, descent and hold. Equations for determining significant parameters of all parts are derived and differences between idealized calculations (based on mathematical performance models) and real ones (based on aircraft flight test data) are explained.

A computer program has been written as a macro in *Lotus 1-2-3*, with data obtained during flights. In the main report simple flowcharts are given to illustrate the methods used. The program results show the required fuel and the time for an airliner of a certain weight performing a mission with a certain range. In the appendix all data and the flowcharts are provided.

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Diplomarbeit in compliance with § 21 of "Ordnung der staatlichen Zwischen- und Diplomprüfung in den Studiengängen Fahrzeugbau und Flugzeugbau an der Fachhochschule Hamburg"

Background

Performance data acquired by flight testing is used by aircraft manufacturers to produce a Performance Engineers Manual (PEM). The PEM contains the basic airplane aerodynamic and engine performance data in graphical and tabular form and may subsequently be used to calculate critical performance parameters, such as climb rate, take-off distance and range.

Task

Starting with the PEM tables for a generic twin engine jet transport aircraft, a user friendly, aircraft performance model is to be generated. A spreadsheet using macros and lookup tables (containing all relevant aerodynamic and engine performance data) is to be developed that will facilitate the user to compute the trip fuel (and hence the brake-release weight) and the trip time for a user specified range. Standard ICAO International flight reserves are to be used for the baseline calculations. It shall also be possible to calculate the range for a given payload, fuel quantity and brake-release weight. The model shall be flexible and shall facilitate the user to study the impact on the fuel burn due to changes in en-route drag, for example.

The results have to be documented in a report. The report has to be written in a form up to internationally excepted scientific standards. The application of the German DIN standards is one acceptable method to achieve the required scientific format.

Erklärung

Ich versichere, daß ich diese Diplomarbeit ohne fremde Hilfe selbständig verfaßt und nur die angegebenen Quellen und Hilfsmittel benutzt habe. Wörtlich oder dem Sinn nach aus anderen Werken entnommene Stellen sind unter Angabe der Quellen kenntlich gemacht.

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Datum Unterschrift

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Nomenclature

a	speed of sound
a	linear acceleration
A	aspect ratio
b	wing span
c	specific fuel consumption (SFC)
c_L	lift coefficient
c_D	drag coefficient
D	drag force
e	Oswald efficiency factor
E	lift-to-drag ratio
g	acceleration due to gravity
h	height
h_p	pressure height
m	aircraft mass
m_F	fuel mass (onboard)
M	Mach number
p	pressure
q	dynamic pressure
Q	mass of fuel burned per unit time
r_a	specific air range (also SAR)
R	range
R	gas constant
s	ground distance
S	wing reference area
t	time
T	thrust
T	temperature
v	velocity, true airspeed (TAS)
v_E	equivalent airspeed
v_V	vertical component of TAS
W	weight (force)
x	still air distance

Greek

α	angle of attack (also AOA)
g	climb angle
d	relative pressure
q	relative temperature
r	density
s	relative density

List of abbreviations

AOA	Angle of attack
CAS	Calibrated airspeed
EAS	Equivalent airspeed
FAR	Federal Aviation Regulations
FL	Flight Level
PEM	Performance Engineers Manual
ICAO	International Civil Aviation Association
ISA	International Standard Atmosphere
OEW	Operational Empty Weight
ROC	Rate of climb
ROD	Rate of descent
SAR	Specific air range
SFC	Specific fuel consumption

1 Introduction

1.1 Motivation

Air traffic is constantly increasing. In the last 30 or 40 years airplanes have become means of transport like trains or cars and almost everyone in the industrial states is able to use them. Never has it been so cheap. In the meantime they have reached gigantic dimensions. The bigger an aircraft is, the more economical it is to use. Compare a DC-3 from the 1940s with a Boeing 747 or even an A3XX, which may be built soon. Such aircraft consume huge amounts of fuel. A Boeing 747-400 can carry more than 168 tons of fuel, almost half of its maximum start weight **(Boeing 2000)**, and its rate of burned fuel per person per 1000 km almost is similar to that of an ordinary car.

Air pollution due to aircraft emissions is a concern. The high altitude operations of jet aircraft makes it even worse. Normal jet airliners operate at heights between ten and thirteen kilometers. Concorde flies at altitudes of up to 20 kilometers. Two substances developed during the burning process are Nitrogenoxide and Carbondioxide. The former is one of the gases which is responsible for damaging the ozone layer. Since the ozone layer is in a region of fifteen to thirty kilometers, the emission of such gases at high altitudes bring them directly into it. Carbondioxide is the most important gas regarding the greenhouse effect, and aircraft produce it in large quantities **(Hagen 2000**).

The limited resources of oil are another factor. The only thing known at the moment to replace kerosine with another fuel could be Hydrogen. The German DASA and the Russian companies Tupolev and Kusnetzow are doing studies about using liquid Hydrogen. The use of this fuel is problematic. It must be kept at a very low temperature. The fuel is also volumous, thus making it impossible to use the wings as tanks (**DASA 2000**).

Of course it has always been a goal to make aircraft more economical. Generally there are three ways to achieve this:

- increase the efficiency of the engines
- reduce the weight of the aircraft
- reduce the drag of the aircraft

Modern jet engines burn much less fuel than old ones. Mainly this is achieved by increasing the bypass ratio. The bypass ratio defines how much air passes the core of the engine and how much is used to burn the fuel. The higher the bypass ratio, the more efficient the engine is. In the times of the oil crisis and high fuel prices research was performed with so called unducted fan engines. These are similar to propeller engines and are very efficient.

Efforts to reduce the weight of an aircraft are mainly limited by the properties of the materials used. The structure of an airplane is still mainly built of aluminum alloys. Newer airplanes have parts consisting of carbon fibre composite, and the importance of these materials is growing, since they can reduce the weight by a significant factor.

Today's attempts to reduce the aerodynamic drag of an aircraft are very complicated. The shape of a normal airliner has been optimized over the years for this purpose. Nowadays research is being conducted to influence the air flow in minute detail. Modeling the surface of the aircraft like that of a shark and boundary layer suction are examples of these drag reduction techniques.

The wings produce the greatest part of the total drag of an aircraft during the cruise. An Airbus A340-300 gets approximately 70 % of its drag from the wing, and only about 22 % is developed by the fuselage **(Mertens 1999)**. Therefore most research is done with regard to the wing. Almost all techniques use additional equipment to reduce drag. However, this equipment increases the weight, which in turn means, less payload can be carried.

1.2 Aim of this work

In order to be able to judge the advantages of drag reduction methods, a computer program was developed to calculate the time and the burned fuel during a mission of a typical airliner. This mission may depend on things like the cruise altitude, speed, range and of course on weight of the airplane. For the fuel reserves the requirements of the International Civil Aviation Organisation (ICAO) have been considered.

Performance data which describe exactly the properties of the aircraft during the flight, are essential. The following are examples of variables integral to the process; the drag polar, the thrust of the engines and the fuel consumption. The data that have been used are for a generic twin engine jet transport aircraft obtained during flights.

The data are provided in the Performance Engineers Manual (PEM) and have been transferred into the software. The program is written as a macro on a spreadsheet in *Lotus 1-2-3*. It consists of one main, which controls several subroutines for the individual parts of the mission. All significant parameters which influence the mission can be entered by the user.

2 Theory

2.1 International Standard Atmosphere

The atmospheric properties in the northern hemisphere have been measured over a long period of time. Based on those measurements, average data were obtained and used to establish the so called International Standard Atmosphere (ISA). The ISA gives an approximation of the conditions in the temperate latitudes of Europe and Northern America. A standard is necessary to have as a basis for estimating and comparing airplane and engine performance.

There are modifications to the ISA, the Tropical Maximum and Arctic Minimum Standard Atmospheres, but they will not be considered in this report.

The ISA is defined as follows:

The temperature at sea level T_0 amounts to $15\,°\mathrm{C}$ (288.15 K). It decreases with every 1000 m by $6.5\,°\mathrm{C}$ up to 11000 m, the end of the Troposphere, which is the Tropopause. In the Stratosphere, the region above the Tropopause, no more cooling takes place and the temperature has a constant value of $-56.5\,°\mathrm{C}$. The sea level pressure is $p_0 = 101325\,\mathrm{N/m^2}$, and the density of the air $r_0 = 1.225\,\mathrm{kg/m^3}$. **(Young 1999) (Boeing 1989)**

The single values depend on each other and the Equation of State describes the relationship between them:

$$\frac{p}{r} = R\,T \tag{2.1-1}$$

The constant R is called the gas constant: $R = 287.053\ \mathrm{J/(kg\cdot K)}$

It is convenient to use in calculations the ratios of the sea level values :

the relative density: $$s = \frac{r}{r_0} \tag{2.1-2}$$

the relative pressure: $$d = \frac{p}{p_0} \tag{2.1-3}$$

the relative temperature: $$q = \frac{T}{T_0} \tag{2.1-4}$$

By using the equation of state for sea level and another arbitrary altitude:

$$\frac{p_0}{r_0} = R\,T_0 \qquad \text{and} \qquad \frac{p}{r} = R\,T \tag{2.1-5}$$

and then dividing the one by the other one obtains:

$$\frac{r}{r_0} = \frac{p/p_0}{T/T_0} \qquad \text{or} \qquad s = \frac{d}{q} \tag{2.1-6}$$

The pressure height h_p at a point in any non standard atmosphere means the altitude in the standard atmosphere with the same pressure. For air traffic the air space is divided into *flight levels* (FL), where pressure heights are measured in feet. The interval between these flight levels amounts to 2000 feet. For example, FL 370 is at a pressure height of 37000 feet, and FL 390 at 39000 feet. A pilot does not know his real altitude. By using flight level 350, he may actually be at 36000 ft on a given day, depending on the pressure. But because all aircraft fly under the same conditions, there is no danger of collision. On the other hand, for operations near ground level like take-off and landing there is the necessity to know the exact height. For these cases the altimeter in the cockpit can be set for different conditions. **(Young 1999)**

Temperature and temperature ratio

- at or below the Tropopause:

$$T\,(^\circ C) = 15 - 0.0019812\ h_p \tag{2.1-7}$$

$$T\,(^\circ K) = 288.15 - 0.0019812\ h_p \tag{2.1-8}$$

$$q = \frac{288.15 - 0.0019812\,h_p}{288.15} \tag{2.1-9}$$

- above the Tropopause:

$$T = -56.5^\circ C = 231.65^\circ K \tag{2.1-10}$$

$$q = \frac{216.65}{288.15} = 0.7519 \tag{2.1-11}$$

Pressure ratio

- at or below the Tropopause:

$$d \;=\; \left(\frac{288.15 \;-\; 0.0019812\, h_p}{288.15}\right)^{5.25588} \qquad (2.1\text{-}12)$$

- above the Tropopause:

$$d \;=\; 0.22336\, e^{\left(\frac{36089.24 - h_p}{20805.1}\right)} \qquad (2.1\text{-}13)$$

Total temperature ratio:

$$q_T \;=\; q\,\left(1 \;+\; 0.2\; M^2\right) \qquad (2.1\text{-}14)$$

Total pressure ratio:

$$d_T \;=\; d\,\left(1 \;+\; 0.2\; M^2\right)^{3.5} \qquad (2.1\text{-}15)$$

(Boeing 1989)

2.2 Airspeeds

Some of the different airspeeds used in this report:

Ground speed: The velocity of the airplane relative to the ground.

True airspeed (TAS, v): The velocity of the airplane relative to the surrounding air. If the surrounding air is moving, as is mostly the case (due to the Jetstream for example), it is unequal to the ground speed.

Equivalent airspeed (EAS, v_E): The velocity the airplane would have at sea level when developing the same dynamic pressure :

$$q \;=\; \frac{1}{2}\, r_0\, {v_E}^2 \;=\; \frac{1}{2}\, r\, v^2 \qquad (2.2\text{-}1)$$

$$v_E \;=\; \sqrt{\frac{r}{r_0}}\; v \;=\; \sqrt{s}\; v \qquad (2.2\text{-}2)$$

Calibrated airspeed (CAS, v_C): The airspeed reading on a calibrated air speed indicator, which is corrected for position and instrument error.
(Young 1999) (Boeing 1989)

2.3 Flight profile

If an aircraft is to be dispatched, one has to to know which requirements must be fulfilled. International flights must operate according to the rules of the International Civil Aviation Organisation (ICAO).

ICAO Annex 6-4.3.6.3 **(Boeing 1996**, chapter E, p. 56):

4.3.6.3 Aeroplanes equipped with turbo-jet engines.

4.3.6.3.2 A)When an alternate aerodrome is required:
To fly to and execute an approach, and a missed approach, at the aerodrome to which the flight is planned, and thereafter:

a) To fly to the alternate aerodrome specified in the flight plan; and then
b) To fly for 30 minutes at holding speed at 450 M (1500 FT) above the alternate aerodrome under standard temperature conditions, and approach and land; and
c) To have an additional amount of fuel sufficient to provide for the increased consumption on
the occurrence of any of the potential contingencies specified by the operator to the satisfaction of the state of the operator.
(Typically a percentage of the trip fuel - 3 % to 6 %).

Therefore the aircraft's flight profile may be divided into several parts. (Figure 2.1)

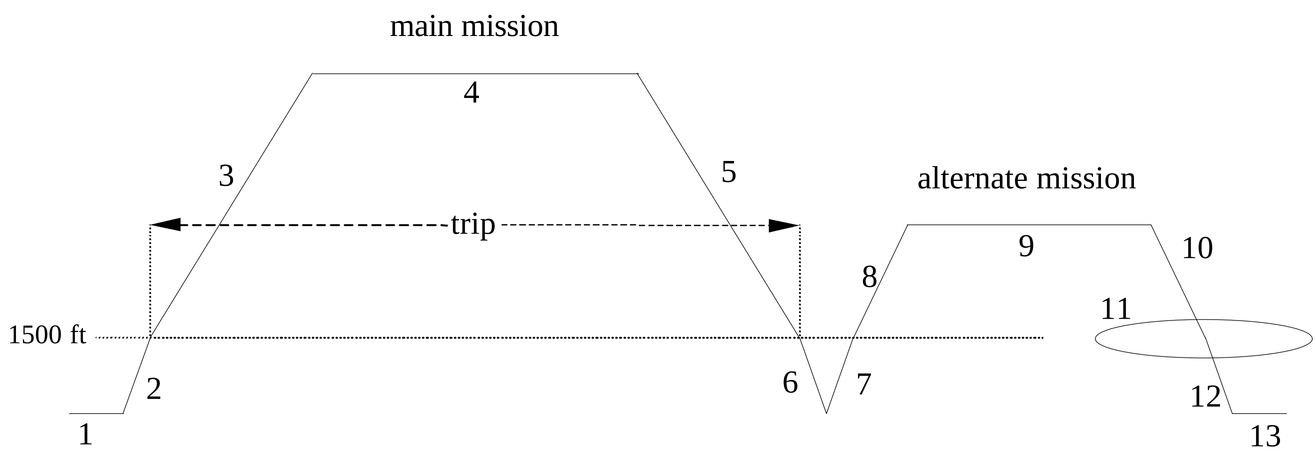


Figure 2.1 Complete mission of an aircraft

1. Engine run-up and taxi to the end of the runway
2. Take-off and climb to 1500 ft
3. Climb to cruise altitude
4. Cruise
5. Descent to 1500 ft
6. Approach
7. Overshoot, climb to 1500 ft
8. Climb to cruise altitude of alternate
9. Alternate cruise
10. Descent to 1500 ft
11. Hold for 30 minutes
12. Approach and land

An alternate airport is needed if for any reason the aircraft cannot land at its designated airport. Runways could be closed for numerous reasons, e.g. bad weather or emergency landings. When choosing an alternate airport various items have to be considered:

- Size and surface of the runway
- Hours of operation, lighting
- Facilities
- Fire fighting, rescue equipment **(Boeing 1996**, chapter E, p. 63)

In the event of a lot of traffic at the alternate airport and permission to land is denied, the airplane must then be able to hold for 30 minutes at an altitude of 1500 ft.

When the mission is known, the required amount of fuel can be determined. For this purpose airlines have computer programs, supplied by the aircraft manufacturers, to calculate the fuel needed for missions. This depends on factors such as the Mach number during the cruise, the distance, the altitude, the payload, and the weather forecast and also the airline specified conditions. Some cargo airlines for instance want to reach their destinations as quickly as possible. They worry little about the burned fuel. However, charter airlines care greatly about fuel consumption and worry little about flight duration.
The captain of the airplane always has the final say as to whether there is sufficient fuel onboard.

It is essential to know the amount of fuel needed. To carry its own fuel the aircraft has to burn fuel. Every pound too much makes the flight more expensive as the distance increases. About 20 % to 35 % of the onboard fuel is required to carry the fuel, depending on the distance to be flown.

The fuel needed for ground operations depends on the airport. Firstly, it is necessary to start and warm up the engines. Then the plane has to taxi to the end of the runway. At large, busy airports the airplane has to wait longer before being granted permission to take off. This is seldom the case at relatively quiet airports, e.g. Shannon, Ireland. In this instance the amount of fuel is based on previous experience.

The actual climb to cruise altitude starts at 1500 ft above airport altitude. In order to get the fuel from brake release to 1500 ft, tables exists which consider the influences of airport altitude, the weight of the plane and the final speed which has to be reached. These data and the values for the descent and approach from 1500 ft are average values supplied by the manufacturer and are based on experience.

The aircraft has to carry fuel for potential contingencies. The contingency fuel is calculated as a fixed percentage of the trip fuel. To reduce the contingency fuel in order to save fuel it is obviously necessary to reduce the trip fuel. How can this be done without cutting down the range?

Assuming an aircraft is to fly from London to Los Angeles, passing New York, and it is only carrying the amount of contingency fuel necessary for a flight to New York, the pilot can check before reaching this city whether it has been used. If not, the airplane can continue the flight to Los Angeles and the airline has saved fuel. If the fuel has been needed, it has to land in New York and refuel for the remaining trip to Los Angeles. Refueling in New York is undesirable, because it is obviously more expensive than without considering redispatching. The former occurs much more often than the latter. Therefore overall fuel consumption is reduced. Figure 2.2 shows this graphically. Los Angeles is the destination and the contingency fuel increases with the distance the airliner has to fly, but because it was redispatched to New York, there is no contingency fuel needed for the last part of the mission. (**Boeing 1996**, chapter E, p. 57-61)

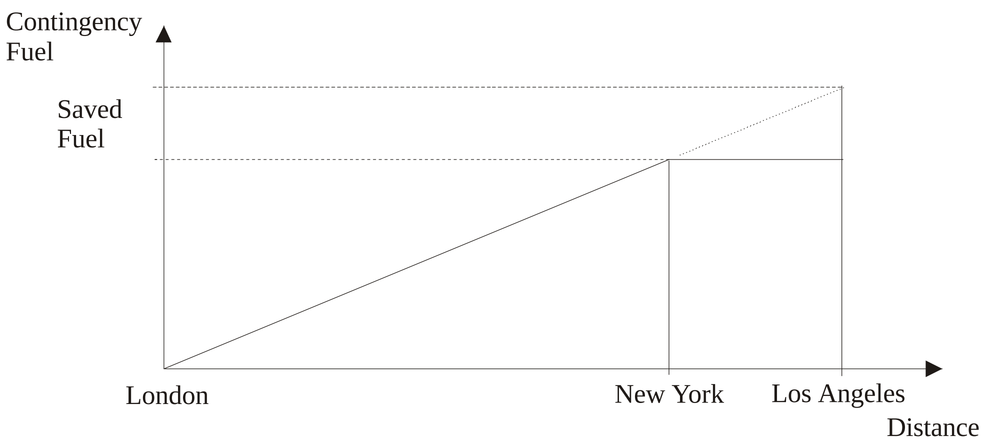


Figure 2.2 Redispatching

2.4 Level Flight

2.4.1 Basic information

During a straight, level and unaccelerated flight the produced lift is exactly equal to the weight of the airplane and the engine thrust equal to the aerodynamic drag. If the pilot increases the thrust the aircraft is able to either climb or accelerate in level flight or both. If the thrust is reduced it will lose height or speed.

The equation for the lift coefficient is given by

$$c_L = \frac{L}{\frac{1}{2}\ r\ S\ v^2} \quad . \tag{2.4-1}$$

Where: L is the lift
S is the wing reference area

The lift can be mostly replaced by the weight of the airplane. In addition to this equation, c_L depends also on the angle of attack a . (Figure 2.3)

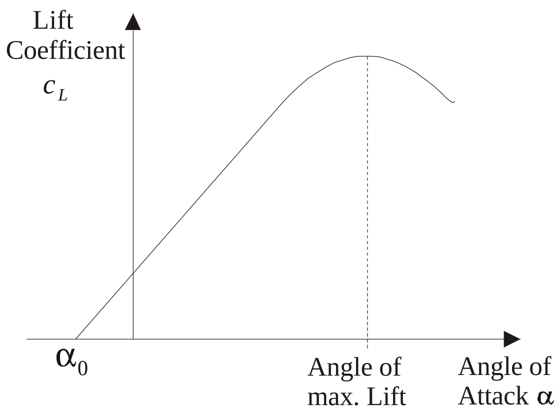


Figure 2.3 Lift coefficient versus angle of attack

The connection between lift and drag is described with a *drag polar.* (Figure 2.4)

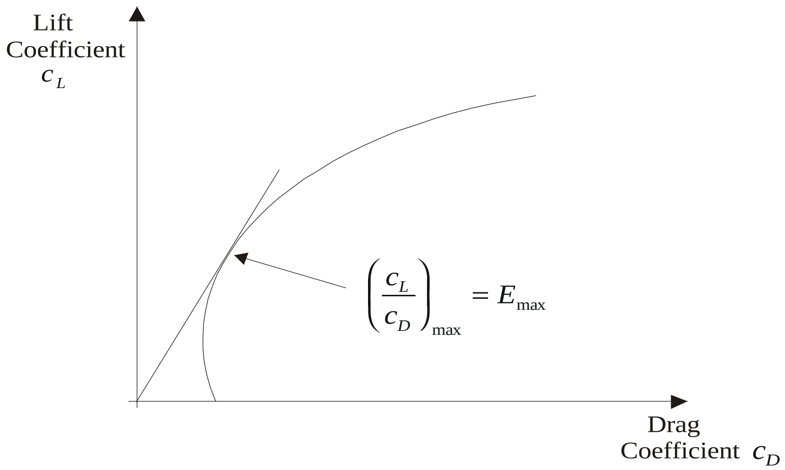


Figure 2.4 Drag polar

The ratio c_L/c_D is called E. $E_{\max}$ is the point with the most lift for the least drag and represents the most efficient operation of the airfoil.

The drag coefficient can be expressed with sufficient accuracy by a parabolic equation of the form:

$$c_D = c_{D_0} + \frac{c_L^2}{p\ A\,e} \qquad (2.4\text{-}2)$$

Where: A is the aspect ratio
e is the Oswald efficiency factor

c_{D_0} is the zero-lift drag coefficient, that means it is equal to c_D when no lift is produced, therefore it is independent of the lift. The second component is the lift-dependent drag and describes mainly the induced drag due to trailing vortexes.

The drag depends on the Reynolds number and also the Mach number. The Reynolds number effects are seen at high angles of attack, which in this study are relatively low for all phases of the flight, and so they will be ignored.

The Mach number is more important. At high Mach numbers, shock waves occur on the airfoil due to compressibility effects. As air is flowing over an airfoil it gets faster and may locally reach speeds greater than the speed of sound. If this happens, a shock wave is produced. The sudden rise in pressure through the shock wave causes a separation of the airflow, which leads to a rise in drag. The associated Mach number is called the *Drag Rise Mach Number*, M_{DR}. When staying below that speed the parabolic description of the drag polar is a very convenient way to do quick calculations.

By plotting the drag coefficient against the Mach number at a given c_L, the rapid drag rise can be noticed at higher Mach numbers due to effects of appearing shock waves. (Figure 2.5)

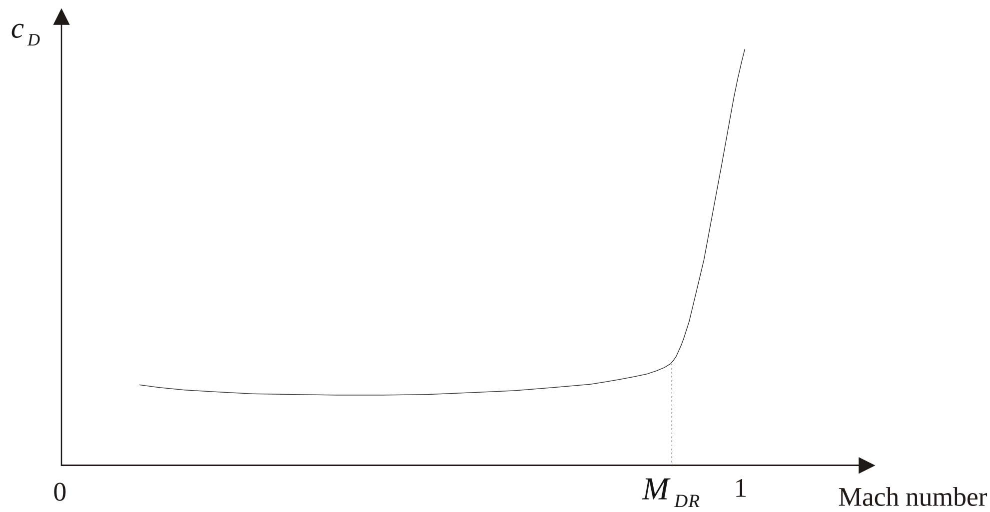


Figure 2.5 Influence of Mach number on drag

However, in practice data are used which are obtained by flight tests, resulting in several polars for several Mach numbers. (Figure 2.6)

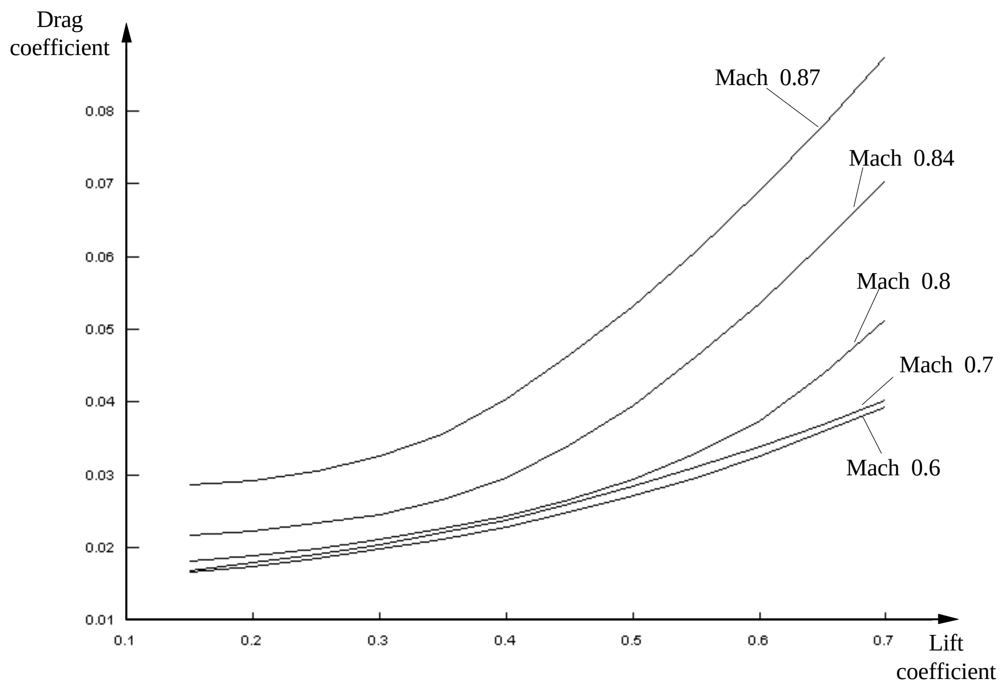


Figure 2.6 Drag polars for several Mach numbers

2.4.2 Range

An aircraft engine responds basically to atmospheric pressure and temperature. Above the Tropopause the temperature is constant in the standard atmosphere, so the thrust varies directly with the ambient pressure. That means, if the altitude is increased, the pressure drops and therefore the thrust, and the ratio T/d theoretically remains constant. Since many airliners operations occur above the Tropopause, T/d is a useful parameter.

By using $\quad v = M\,a \quad$ with $\quad a = a_0 \sqrt{q}$

and $\quad d = s\,q$

and $\quad s = \dfrac{r}{r_0}$,

Equation [2.4-1] may be written as

$$L = c_L \frac{1}{2} r_0 \; S \; d \; M^2 \; a_0^2 \tag{2.4-3}$$

or

$$\frac{L}{d} = c_L \frac{1}{2} r_0 \, S \, M^2 \, a_0^2 = \frac{W}{d} \tag{2.4-4}$$

Thus,

$$\frac{D}{d} = c_D \frac{1}{2} r_0 \, S \, M^2 \, a_0^2 = \frac{T}{d} \tag{2.4-5}$$

The thrust per engine is given by the manufacturer in terms of F_N/d, net thrust over delta, depending on the Mach number and the altitude.

Information about the fuel consumption is provided either in *Thrust Specific Fuel Consumption* (TSFC) or in *Corrected Fuel Flow* (CFF) formats. TSFC is given the symbol *c* and means the burned fuel per unit time per unit thrust. It is a function of altitude, Mach number and thrust and for estimations it is often assumed to be constant for the idealized powerplant in the cruise region. The lower *c*, the more efficient is the engine.

Data for this report were given in tables as Corrected Fuel Flow, depending on altitude, Mach number and net thrust per engine. It is the fuel flow, burned fuel per unit time per engine, generalized by q and d:

$$\text{corrected fuel flow} = \frac{\text{fuel flow}}{d_T \ q_T^{0.6363}} \qquad (2.4\text{-}6)$$

where: d_T is the total pressure ratio $\frac{p_T}{p_0}$

$q_T^{0.6363}$ is the total temperature ratio $\frac{T_T}{T_0}$ to the 0.6363 power

(Boeing 1996, chapter D, 2.16-2.18)

The still air distance flown per unit fuel burned is called *Specific Air Range* (SAR) and given the symbol r_a. An aircraft achieves its maximum range by flying at the condition for maximum SAR.

$$r_a = -\frac{dx}{dm_F} \qquad (2.4\text{-}7)$$

Where: x is the still air distance
m_F is the onboard fuel mass

Because the change in weight is negative the term has a negative sign. Dividing numerator and denominator by dt gives:

$$r_a = \frac{\frac{dx}{dt}}{-\frac{dm_F}{dt}} \qquad (2.4\text{-}8)$$

The term dx/dt may be written as velocity. The denominator represents the fuel burned per unit time by all engines and is given the symbol Q.

$$r_a = -\frac{dx}{dm_F} = \frac{dx/dt}{-dm_F/dt} = \frac{v}{Q} \qquad (2.4\text{-}9)$$

$$dx = -r_a \ dm \qquad (2.4\text{-}10)$$

$$R = \int dx = -\int r_a \ dm = -\int \frac{v}{Q} dm \qquad (2.4\text{-}11)$$

The fuel flow Q is equal to the Specific Fuel Consumption multiplied by thrust (of all engines):

$$Q = c\,T \tag{2.4-12}$$

Since in unaccelerated level flight the thrust is equal to the drag of the airplane and the weight is equal to the produced lift. The equation can be written as follows:

$$Q = c\frac{D}{L}W = c\frac{D}{L}m\,g = c\frac{c_D}{c_L}m\,g \tag{2.4-13}$$

Hence:

$$R = -\int \frac{v\ c_L}{c\ g\ c_D}\frac{dm}{m} \tag{2.4-14}$$

There are different ways to solve this integral. By assuming the Specific Fuel Consumption is constant during the flight (which can be done for estimations), there are three types of cruise to handle the other factors:

1. Flight at constant altitude and constant lift coefficient
2. Flight at constant airspeed and constant lift coefficient
3. Flight at constant airspeed and constant altitude

The second flight schedule gives the greatest possible range. As v and c_L, therefore c_L/c_D, are constant, is the result:

$$R = -\frac{v\ c_L}{c\ g\ c_D}\int\frac{dm}{m} = \frac{v\ c_L}{c\ g\ c_D}\ln\left(\frac{m_1}{m_2}\right) \tag{2.4-15}$$

This is the *Breguet Range Equation*. The equation of the lift coefficient [from 2.4-3]

$$c_L = \frac{W/s}{\frac{1}{2}\,r_0\ S\ v^2} \tag{2.4-16}$$

shows that the term W/s has to be constant under these circumstances. While fuel is burned and the weight drops, s has to drop as well. This is achieved if the aircraft is allowed to climb, which happens very slowly. During the whole cruise the pilot has only to ensure a constant true airspeed.

Now, if all airliners were consistent in changing their altitude to achieve the greatest possible range, it would lead to chaos in air traffic. In reality an aircraft remains, for a certain time, at its given altitude and may then - if it is allowed to - climb to another flight level. This only makes sense if it stays long enough at the higher altitude to save more fuel than was burned while climbing. It is called a Step Climb. **(Young 1999**)

2.4.3 Hold

The flight condition permitting an aircraft to achieve its greatest range is different to the condition which makes it possible to be airborne for the longest time. For instance, Coast Guard airplanes on a search mission have to be able to fly a certain distance to the place of the accident and then circle there for as long as possible. Commercial aircraft often have to wait before they get permission to land. Therefore the plane has to fly at conditions for lowest fuel consumption per unit time.

The fuel consumption per unit time is given by:

$$Q = -\frac{dm_F}{dt} \tag{2.4-17}$$

$$Q = c\,T = c\,D = c\,\frac{D}{L}W = c\,\frac{c_D}{c_L}m\,g \tag{2.4-18}$$

When the airplane flies at the speed of the least drag it will achieve its maximum endurance, i.e. the Specific Fuel Consumption is assumed to be constant for estimations.

$$t = -\int \frac{1}{Q}\,dm = -\frac{1}{c\,g}\int \frac{c_L}{c_D\,m}\,dm \tag{2.4-19}$$

Again there are different ways to solve the integral:

- flight at constant altitude and constant lift coefficient
- flight at constant airspeed and constant lift coefficient
- flight at constant altitude and constant airspeed

On a flight with constant lift coefficient the endurance time is given by:

$$t \;=\; \frac{c_L}{c_D\; c\; g} \ln \frac{m_1}{m_2} \tag{2.4-20}$$

2.4.4 Integrated Range and Integrated Time

However, to do more exact calculations the specific fuel consumption can not be assumed to be constant. In this study the range is determined following the method of the *Integrated Range*.

The Specific Air Range for a certain height and Mach number is computed using [2.4-9]

$$r_a \;=\; \frac{v}{Q} \quad .$$

In order to get the fuel flow Q, the net thrust per engine F_N/d is obtained from the available thrust. The corrected fuel flow and therefore the fuel flow per engine and Q can be determined, if the height, the Mach number and the net thrust are known. Now the Specific Air Range can be plotted against weight. (Figure 2.7)

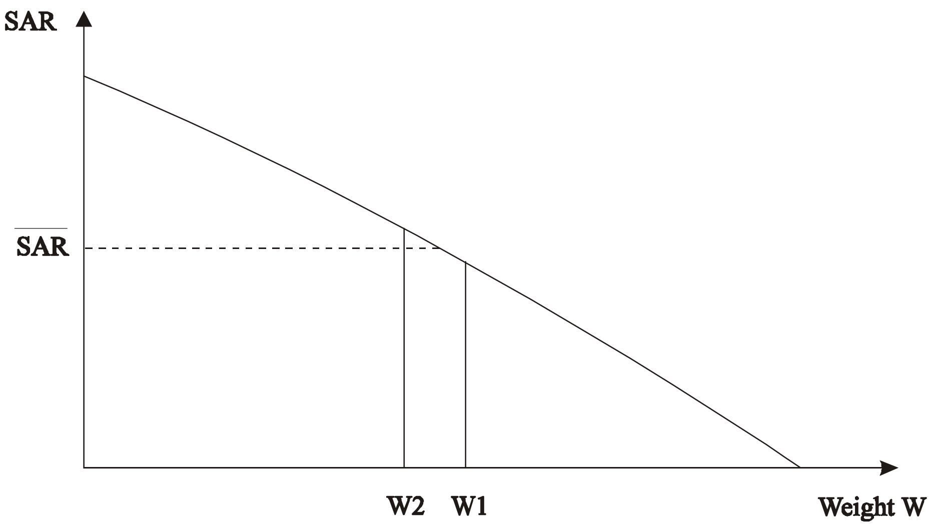


Figure 2.7 Specific Air Range

Every weight interval corresponds to an average Specific Range designated as $\overline{\text{SAR}}$. Multiplying the change in weight W1-W2 with $\overline{\text{SAR}}$ gives the distance the aircraft can fly with the fuel quantity W1-W2. By summing up or integrating the distances resulting from all changes in weight the Integrated Range is obtained (Figure 2.8) **(Lufthansa 1988)**:

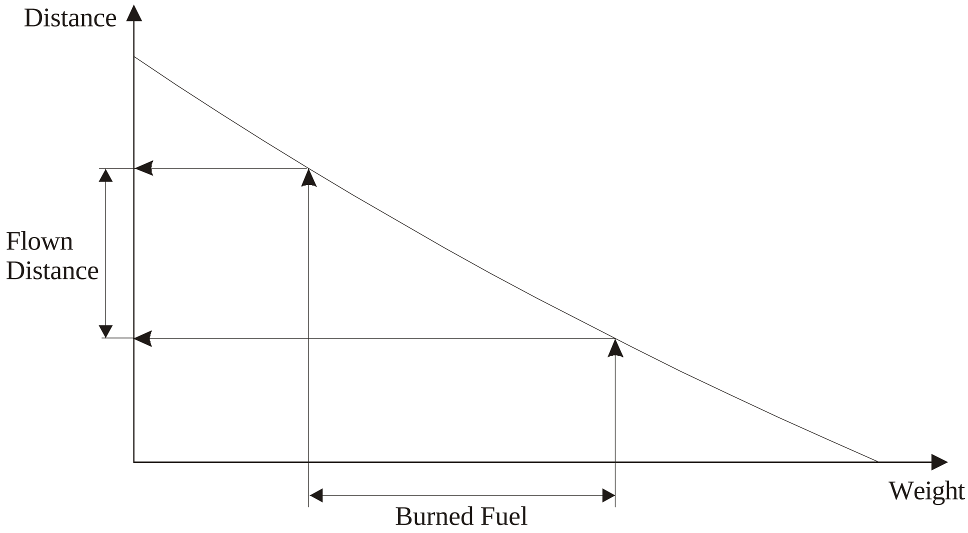


Figure 2.8 Integrated Range

Once the Integrated Range is obtained, it is only a little step to determine the Integrated Time by dividing every calculated distance by the true airspeed. (Figure 2.9)

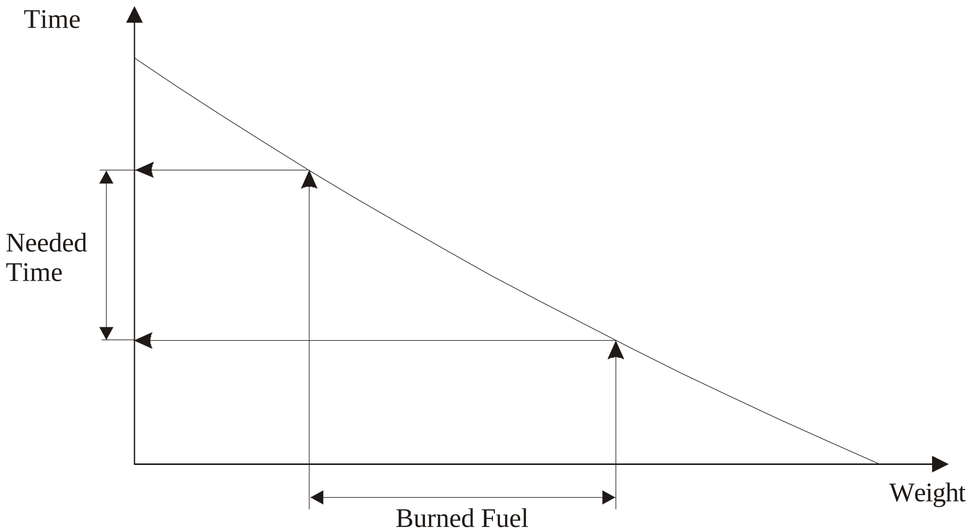


Figure 2.9 Integrated Time

The Integrated Range and the Integrated time are very convenient methods to make flight planning easier and can be used for in-flight planning, checking the fuel and the time needed for diversions etc. **(Lufthansa 1988)**

2.5 Climb

2.5.1 Basic information

There are two terms to describe the climb performance of an aircraft:

- the climb angle g and
- the rate of climb (ROC) v_v

The former measures the gain in height over a certain flight distance. The rate of climb expresses the altitude gain over a certain time.
To derive equations it is convenient to consider first the forces acting on the aircraft during the flight. Figure 2.10 shows an aircraft climbing due to an excess of thrust over drag along a linear path.

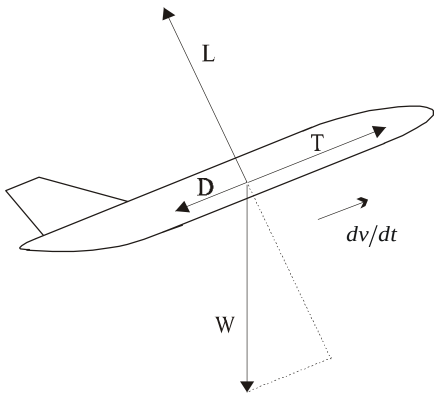


Figure 2.10 Acting forces on an aircraft during the climb

2.5.2 Climb angle

The climb angle g is defined as the angle between the horizontal and the flight path during a climb without any winds. (Figure 2.11)

In the presence of headwinds the flight path angle increases and the distance flown over ground is getting shorter, whilst the climb angle may stay the same. But because headwinds provide a part of the speed necessary to climb, the pilot can use the remaining thrust to lift the nose of the aircraft in order to increase the climb angle and the rate of climb. Therefore airliners always try to take off with headwinds, so they are gain a safe height in spite of a shorter flown distance and a smaller period of time. Tailwinds have a reverse influence on the climb and so they are mostly unwanted.

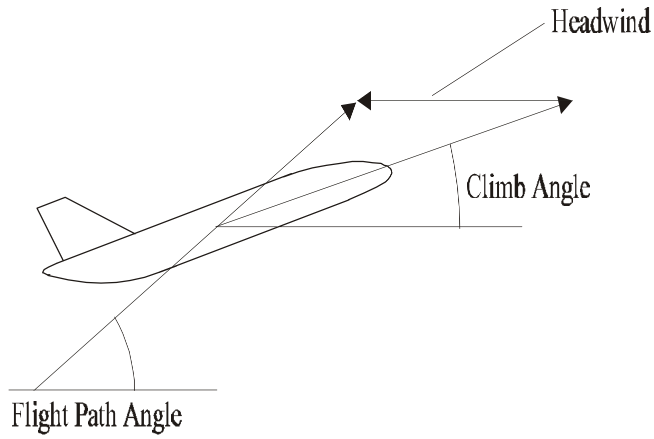


Figure 2.11 Influence of headwind on climb angle

Generally the climb angle of airliners is small (i.e. less then about $15°$), so that the small angle assumptions can be used. The angle between the line of the thrust and the flight path is much smaller and may be ignored. Considering the equilibrium of the acting forces leads to:

$$T - D - W \sin g = \frac{W}{g}\frac{dv}{dt} \qquad (2.5\text{-}1)$$

and
$$L - W \cos g = 0 \qquad (2.5\text{-}2)$$

The term $\frac{W}{g}\frac{dV}{dt}$ is the force due to the mass of the aircraft multiplied by the acceleration in direction of the flight path.

From [2.5-1] the angle of climb may be expressed as:

$$\sin g = \frac{T - D}{W} - \frac{1}{g}\frac{dV}{dh}\frac{dh}{dt} \qquad (2.5\text{-}3)$$

The vertical component of the airspeed may be written as:

$$v_v = v \sin g = \frac{dh}{dt} \qquad (2.5\text{-}4)$$

Substituted in [2.5-3]:

$$\sin g = \frac{T - D}{W} - \frac{v}{g}\frac{dv}{dh}\sin g \qquad (2.5\text{-}5)$$

Hence:

$$\sin g = \frac{\frac{T}{W} - \frac{D}{W}}{1 + \frac{v}{g}\frac{dv}{dh}} \tag{2.5-6}$$

And with:

$$W \cos g = L \approx W \tag{2.5-7}$$

$$\sin g = \frac{\frac{T}{W} - \frac{D}{L}}{1 + \frac{v}{g}\frac{dv}{dh}} = \frac{\frac{T}{W} - \frac{c_D}{c_L}}{1 + \frac{v}{g}\frac{dv}{dh}} \tag{2.5-8}$$

The climb gradient is usually expressed as $\tan g$ and represents the gain in height with respect to the horizontal distance flown in the absence of any winds. For example, when the climb gradient is 0.3, the aircraft gets every 1000 ft flown 300 ft higher. Usually the value is given in percent. In this case it is 30 %.

Using the small angle approximation:

$$g \approx \tan g \approx \sin g = \frac{\frac{T}{W} - \frac{c_D}{c_L}}{1 + \frac{v}{g}\frac{dv}{dh}} \tag{2.5-9}$$

In the case of a steady, unaccelerated flight, the change of speed with respect to height is zero and thus the equation becomes:

$$g \approx \tan g \approx \sin g = \frac{T}{W} - \frac{c_D}{c_L} \tag{2.5-10}$$

2.5.3 Rate of climb

The rate of climb is equal to the vertical speed component and is given by equations [2.5-4] and [2.5-6]:

$$v_v = v \sin g = \frac{\dfrac{T}{W} - \dfrac{c_D}{c_L}}{1 + \dfrac{v}{g}\dfrac{dv}{dh}} v \qquad (2.5\text{-}11)$$

The acceleration factor describes the change in speed during the climb:

$$f_{acc} = \frac{v}{g}\frac{dv}{dh} \qquad (2.5\text{-}12)$$

In case of a climb being flown at constant true airspeed, the acceleration factor becomes zero and equation [2.5-11] can be written as:

$$v_v = \left(\frac{T}{W} - \frac{D}{W}\right) v \qquad (2.5\text{-}13)$$

The acceleration factor depends on the Mach number, the height and the climb speed condition:

$$f_{acc} = \frac{1.4\ M^2}{2}\, y \qquad (2.5\text{-}14)$$

At a climb at constant

Mach : $y = -a$

EAS : $y = 1 - a$

CAS : $$y = \frac{\left[\left(1 + 0.2\ M^2\right)^{3.5} - 1\right]}{0.7\ M^2 \left[\left(1 + 0.2\ M^2\right)^{2.5}\right]} - a \qquad (2.5\text{-}15)$$

Where $a = 0.190263$ (below the tropopause)

$a = 0$ (above the tropopause)

(Boeing 1989, p. 3.141)

2.5.4 Climb schedule

Airliners are most efficient during the cruise. Therefore they have to reach their cruise level as soon as possible after take off. To get the speed for the best rate of climb several things can be taken into account, each one with a different result:

- shortest time to reach cruise altitude
- shortest total flight time
- lowest fuel consumption on the entire flight
- lowest operating costs
- highest climb angle
- simplicity of flight operation (**Lufthansa 1988)**

In addition there are some conditions of the Federal Aviation Regulations (FAR) to fulfill when considering the climb performance. The true airspeed for the best rate of climb varies with altitude and is partially very close to a constant calibrated airspeed and to a constant Mach number. Depending on the aircraft, climb speed schedules are established either in terms of indicated airspeed (aircraft with pneumatic speed indicators) or calibrated airspeed (electronic speed indicators). **(Boeing 1989**, p. 3.138)

For example:
The climb schedule 250 / 290 / 0.8 means, the aircraft climbs until 10000 ft with a calibrated airspeed of 250 knots. Above this height the speed is 290 knots as long as a Mach number of 0.8 is reached. This Mach number is held until the cruise height is reached.

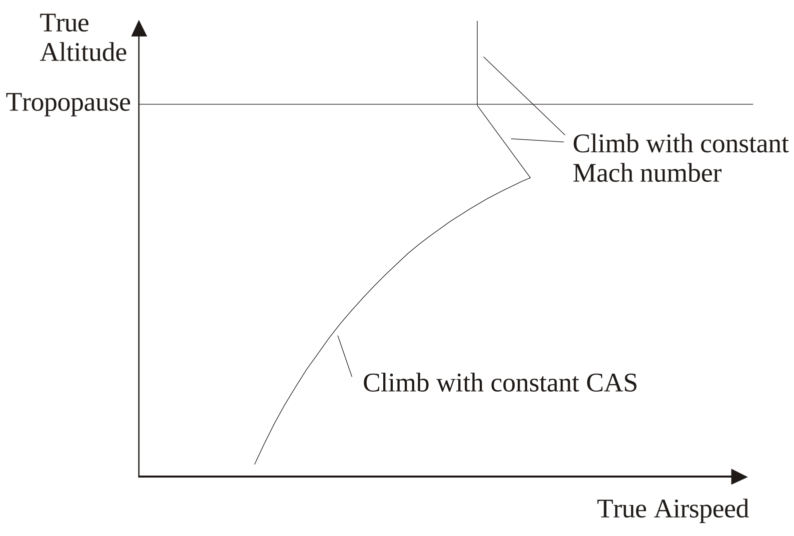


Figure 2.12 Typical airliner climb schedule:
Constant CAS climb, followed by constant Mach number climb
(Boeing 1989, p. 3.139**)**

2.5.5 Time to climb

The vertical motion of an aircraft is a negative accelerated movement, since the rate of climb drops with increasing altitude. A look at the rate of climb shows, that this depends on several factors:

$$v_v = \frac{\frac{T}{W} - \frac{c_D}{c_L}}{1 + \frac{v}{g}\frac{dv}{dh}} v$$

During the flight, as fuel is burned, the airplane loses weight. It loses even more when there is high fuel consumption as a result of the climb. The fuel flow is the product of the specific fuel consumption and the thrust. The former can be considered as constant for the ideal jet, but the latter decreases with dropping density and depends on the throttle setting. The changing in Mach number and density affect the lift coefficient and therefore the drag coefficient.

It is necessary to divide the vertical distance in intervals. At each increment the acceleration may be assumed to be constant, making the change in speed linear:

$$dt = \frac{dh}{v} \qquad (2.5\text{-}16)$$

The smaller the step of each interval, the more accurate the result of the calculation is. It is recommended to reduce the increments at higher altitudes, because of the flown distance. The needed time increases with respect to the change in height, which in turn causes a greater error.

The calculations of the time and of the fuel needed for the climb are connected. Because the time depends on the change in weight, and the change in weight depends on the fuel burnt during a certain period of time. There are several very similar ways to obtain the time to climb, two of them will be described.

First method:

Equation [2.5-16] can be written as:

$$t = \int_{h_1}^{h_2} \frac{1}{v_v}\, dh \qquad (2.5\text{-}17)$$

The change in the vertical speed with respect to height may be shown as follows (Figure 2.13):

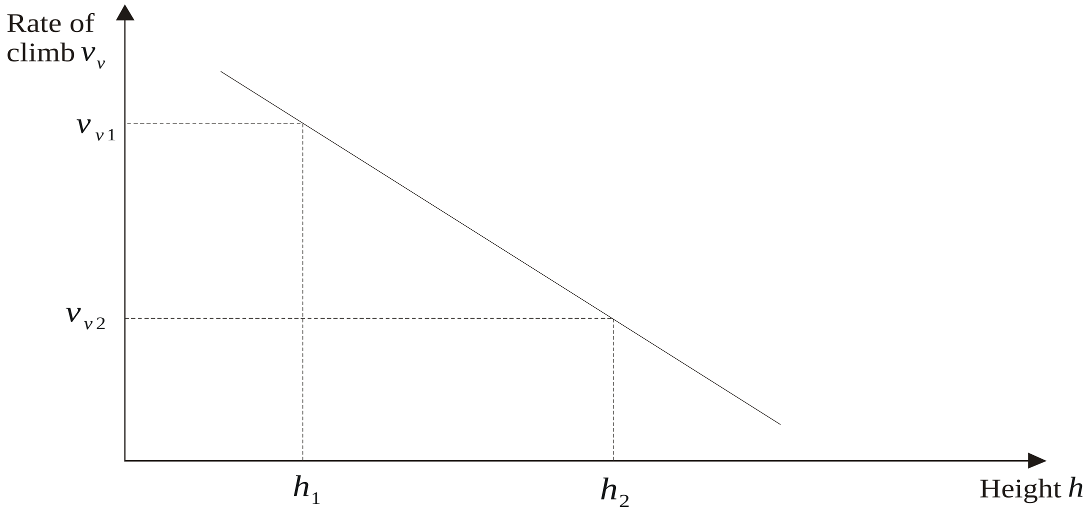


Figure 2.13 Change in vertical speed during the climb (simplified)

The slope of the line is given by:

$$m = \frac{v_{v1} - v_{v2}}{h_2 - h_1} \tag{2.5-18}$$

The equation for the ROC is therefore:

$$v_v = m\left(h - h_1\right) + v_{v1} \tag{2.5-19}$$

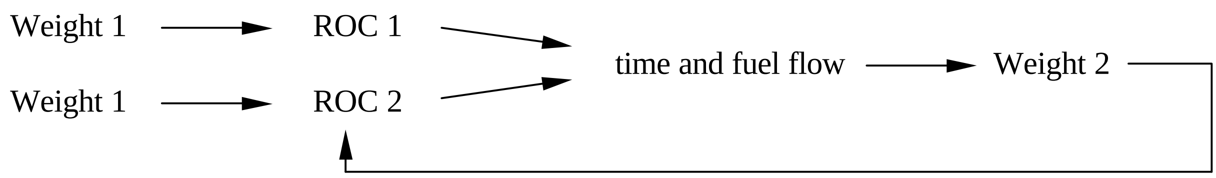
Hence:

$$t = \int_{h_1}^{h_2} \frac{1}{m\left(h - h_1\right) + v_{v1}} = \frac{1}{m} \ln\left(\frac{v_{v2}}{v_{v1}}\right) \tag{2.5-20}$$

The rate of climb v_{v2} at the end of the interval is unknown, because the weight is unknown. By making the assumption that the weight does not change during this step, a rate of climb at point two may be calculated with the weight of point one. Now the time can be obtained.

To ascertain the amount of fuel burnt the fuel flow is needed. As with the rate of climb it does not remain constant during the climb, but depends on Mach number, thrust and height. With given fuel flows at both altitudes an average fuel flow is determined, which is used with the time to compute the change in weight.

Because the guessed weight from this first step is comparatively close to the real weight at altitude 2, this method is good for the use without the help of a computer.



Second method:

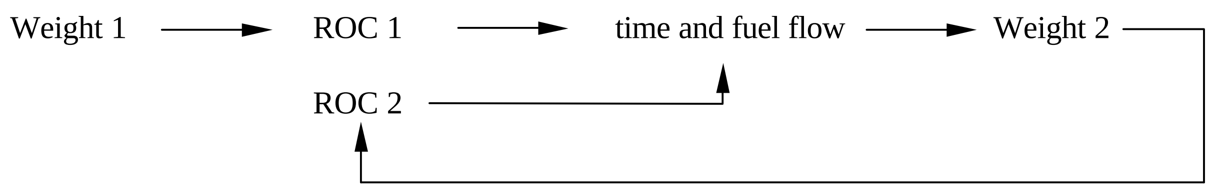
Starting with the equation for the constant accelerated motion:

$$s_2 = \frac{a}{2} t^2 + v\,t + s_1 \tag{2.5-21}$$

and $$v_2 = a\,t + v_1 \tag{2.5-22}$$

gives $$t = \frac{2\left(s_2 - s_1\right)}{v_1 + v_2} = \frac{2\ \Delta h}{v_{v1} + v_{v2}} \tag{2.5-23}$$

The rate of climb is assumed to be constant and therefore, $\mathrm{ROC}_1 = \mathrm{ROC}_2$. The time can be determined and with the average fuel flow the needed fuel and the weight at point 2 is obtained. The second step uses this new weight to get the rate of climb at point 2.



2.6 Descent

The descent is actually the same as the climb, except the angle g is less then zero. Therefore the rate of descent has to be redefined:

$$v_v = -\frac{dh}{dt} = -v \sin g = -\frac{\dfrac{T}{W} - \dfrac{c_D}{c_L}}{1 + f_{acc}}\, v \tag{2.6-1}$$

Generally, it is desirable to start the descent early in order to save fuel, and make a descent with idle thrust. That demands a long distance at the descent. The greatest possible range is achieved while flying at the lowest glide angle:

$$\sin g = -\frac{T - D}{W} = \frac{D - T}{W} = \frac{D - T}{L - W} \qquad (2.6\text{-}2)$$

The acceleration factor has little influence and may be ignored. If the engines run at idle thrust, then sometimes the thrust is even less then zero, that means additional drag is produced. To get the lowest glide angle, the aircraft has to fly at a speed as near as possible to the maximum L/D. It is interesting, that at zero thrust the glide angle is exactly the same for all weights. **(Boeing 1989**, p. 3.210)

3 The program

3.1 Structure

The program is written as a macro in *Lotus 1-2-3*. All data the program uses were available in the Performance Engineers Manuel (PEM) for a generic twin engine aircraft and have previously been transferred into *Lotus 1-2-3*. The data was:

- High Speed Drag Polar (Appendix A.15 - A.16)
- Corrected Fuel Flow (Appendix A.1 - A.8)
- Maximum Climb Thrust (Appendix A.9)
- Minimum Idle In-flight Thrust (Appendix A.10)
- Minimum Idle Fuel Flow (Appendix A.11)
- Fuel and the time from brake release to 1500 ft (Appendix A.12 - A.13)
- Holding speed at 1500 ft (Appendix A.14)

The program computes every part of the mission according to chapter 2.3 *Flight profile*. It consists of a main macro, which controls the subroutines for the cruise/hold and the climb/descent. The order in which they are called depends on the input data entered by the user. They are provided with required data like the drag coefficient or the fuel flow by several little subroutines. Figure 3.1 shows the hierarchy of the individual parts.

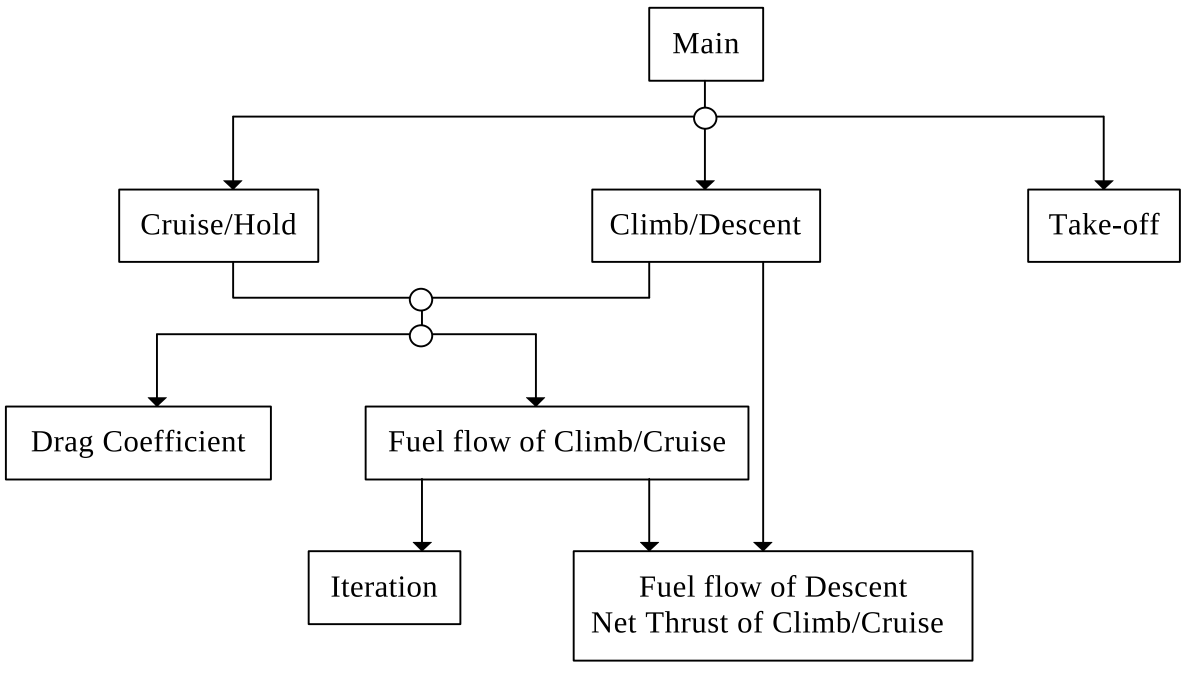


Figure 3.1 Hierarchy of the individual macros

3.2. The main

With the input data, the user defines the exact conditions of the mission and the single weights of the aircraft. The input data for the major mission and the diversion are

- the climb and descent schedule
- the cruise altitude
- the cruise Mach number
- the sizes of the single steps (chapter 3.2.1 and 3.2.2) and
- the range.

The weights of the aircraft the macro is using directly for the calculation are

- the operational empty weight (OEW)
- the onboard fuel weight (including the fuel for engine run-up and taxi)
- the payload and
- the brake release weight.

In addition there are the limit weights which have to be defined, like the maximum take-off weight, the maximum payload and the maximum fuel weight. Note that the latter depends on and changes with the density of the fuel, since the maximum possible amount of fuel is actually limited by the volume of the tanks. Limit weights are used for controlling the input and during the calculation. Figure 3.2 shows the input box of the main macro.

			MAJOR MISSION				DIVERSION	
CLIMB	schedule:	250	290	0.8		250	290	0.52
		stepsize:	2000	ft		stepsize:	2000	ft
		at:	35000	ft		at:	20000	ft
CRUISE		mach :	0.8			mach :	0.52	
		stepsize:	2000	lb		range:	200	nm
						stepsize:	2000	lb
DESCENT	schedule:	0.78	290	250		0.52	290	250
		stepsize:	2000	ft		stepsize:	2000	ft
Oper. empty w.	(lb)	128730			LIMITS	max. take-off weight	(lb)	255000
						max. payload	(lb)	56600
		INPUT	OUTPUT			max. fuel weight	(lb)	77422
Fuel on board	(lb)	70000	70000	run				
Payload	(lb)	50000			ERRORS	contingency fuel	(%)	0.1
Brake release w.	(lb)	0	247830	check		range	(nm)	1
Range	(nm)	0	2954					
Contingency fuel	(%)	4.00	3.99		needed time for calculation :			00:02:53

Figure 3.2 Input box of the main macro

The amount of the contingency fuel which is left in the tanks after touch down may be entered as a percentage of the trip fuel. Since the principle of the calculation is an iteration, there is the necessity to define errors as limits for this iteration. The first absolute error is the contingency fuel error in percentage form. The second concerns the range, an absolute error in nautical miles.

For example; "4" may be entered as the required contingency fuel, means 4 % of the trip fuel, and "1" may entered as the error. Now the real contingency fuel may fluctuate between 3 % and 5 %. The second error has the same influence on the range in miles. The accuracy of the results depends on those errors and on the *stepsizes* (chapter 3.2.1 and 3.2.2). The more accurate the results have to be, the smaller the stepsizes to be chosen, and the longer the runtime.

There are three ways to solve the proposed problem. After starting the program, the input data are analyzed and are then calculated as either the onboard fuel (including the fuel necessary for taxi to the runway), the payload or the range. Data which the user does not know may be set to zero or the cells may remain empty. Table 3.1 shows all possible types of input.

Table 3.1 Types of input

given values	calculated value
fuel, payload OR fuel, brake release weight OR only fuel (no payload) OR brake release weight, payload	range
payload, range	fuel (and hence the brake release weight)
fuel, range	payload (and hence the brake release weight)

Chapter 3.4 gives to every of the input types in table 3.1 an example. Expect for the input and output box, the settings are like in figure 3.2.

It is very difficult to obtain the possible payload for a given distance and a given amount of fuel, because there is no weight at all to start the iteration. The brake release weight is unknown, and the zero fuel weight changes with the searched payload. To be sure, the iteration converges to a final value, the two limit distances have to be determined; first the range for the given amount of fuel and no payload and second, the range for the maximum payload. If the required range is within those two limits, the calculation will be performed.

After performing any calculation, there is the possibility to check the results with another little Macro. Since it contains no iterations, the results of it are slightly different from the other ones and closer to the real ones.

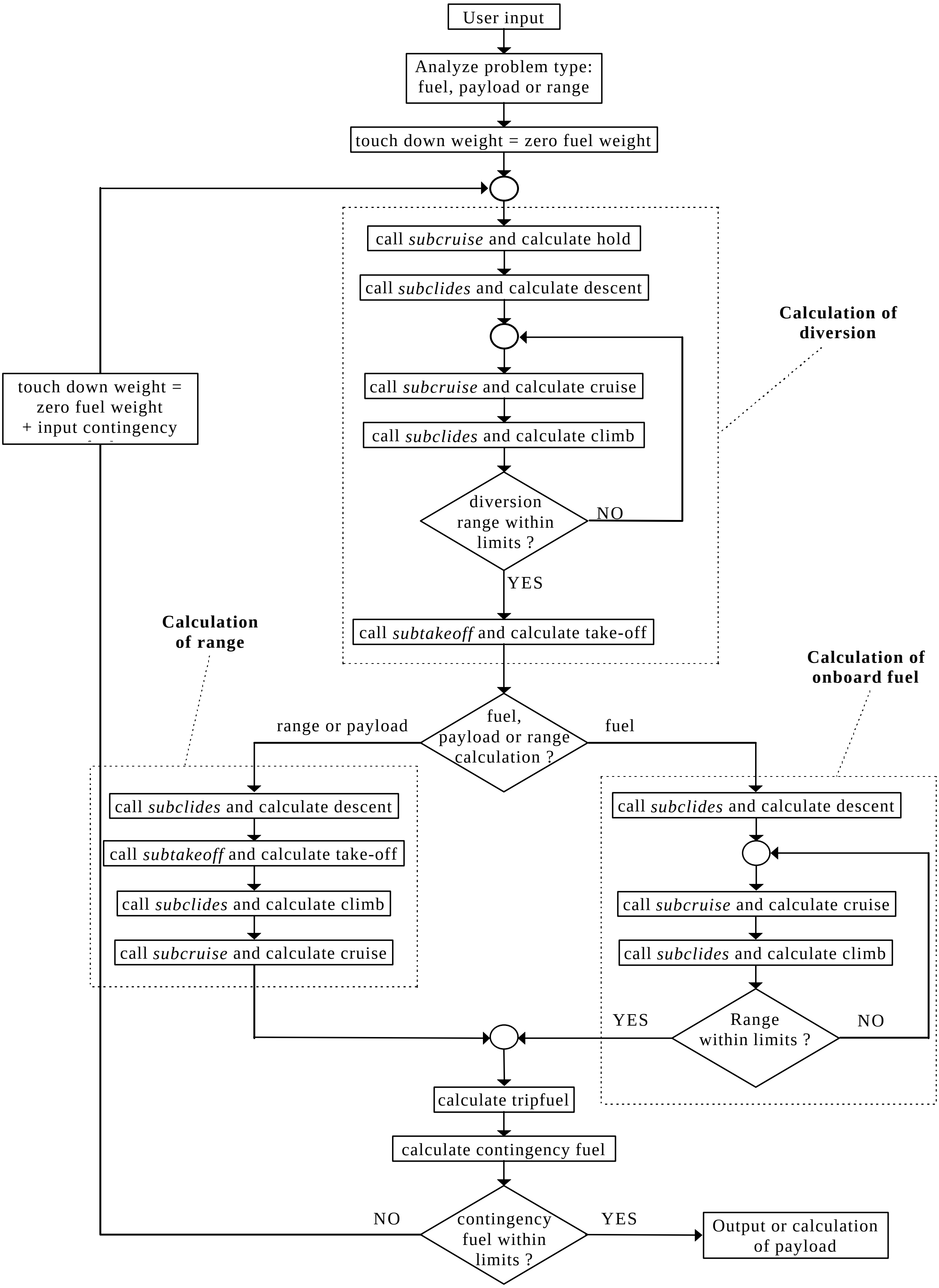


Figure 3.3 Simpified flowchart of the main macro (only range and fuel calculation, without payload branch)

3.2.1 Cruise

The macro for the cruise is called *subcruise*. It is used to calculate a straight, level flight with no change in speed or a hold.
The necessary input values are as follows:

- the altitude (in feet)
- the Mach number
- the stepsize (in pounds)
- one weight, at the start or at the end of the flight (in pounds)
- the information, whether a hold is considered or not

In addition, the following are needed:

- the other weight (in pounds) or
- the distance (in nautical miles) or
- the time

The output values are:

- the two weights and therefore the burned fuel (in pounds)
- the flown distance (in nautical miles)
- the time

The macro is able to run backwards. When the given time or distance is negative, or the first weight is less then the second, the macro recognizes that the considered flight actually goes backwards. In this case the results are negative except the weights. This makes some iterations, required by the main, simpler.

The working principle is based on the method of the Integrated Range (2.4.4). For a given weight all parameters are computed:

- the lift coefficient and the drag coefficient
- the necessary thrust and the net thrust over delta
- the corrected fuel flow and the fuel flow
- the SAR

After determination of the SAR for two weights, the flown distance can be determined according to figure 2.7 and 2.8. By summing up all the single distances the range (and the needed time) is obtained. The difference between those two weights is the *stepsize* in pounds and may be entered by the user. The smaller the steps, the more accurate the calculation, but the longer the runtime. Depending on the entered limit, the macro stops either when reaching the second weight, the range or the time. (Flowcharts C.4 and C.5)

3.2.2 Climb and descent

Climb and descent of a mission will be calculated in one macro, since both are very similar. The only difference between them is the determination of the produced thrust and the fuel flow. The macro is called *subclides*.

The necessary input data are:

- the flight schedule (calibrated airspeed in knots)
- one weight (in pounds)
- both altitudes (in feet)
- the stepsize (in feet)

The output data are:

- the time (in minutes)
- the distance (in nautical miles)
- the burned fuel (in pounds)
- and therefore the unknown weight (in pounds)

The macro recognizes with the flight schedule whether a climb or a descent is to be calculated. The given weight has to be that of the first entered altitude. At the second altitude the weight is always required. Like the cruise-macro it is able to run backwards. For example, a climb has to be calculated and the weight at the top of climb is known. In this case, the first altitude is at the top of climb (TOC), and the second altitude is at 1500 feet. After a backwards calculation the results are negative, except the weight, again like in the cruise-macro.

The macro uses the first method described in chapter 2.5.5. It starts with the determination of the necessary parameters of the first altitude and the next one, which is calculated by adding the stepsize chosen by the user. After that it performs two iterations to obtain the time and the burned fuel between these two heights.

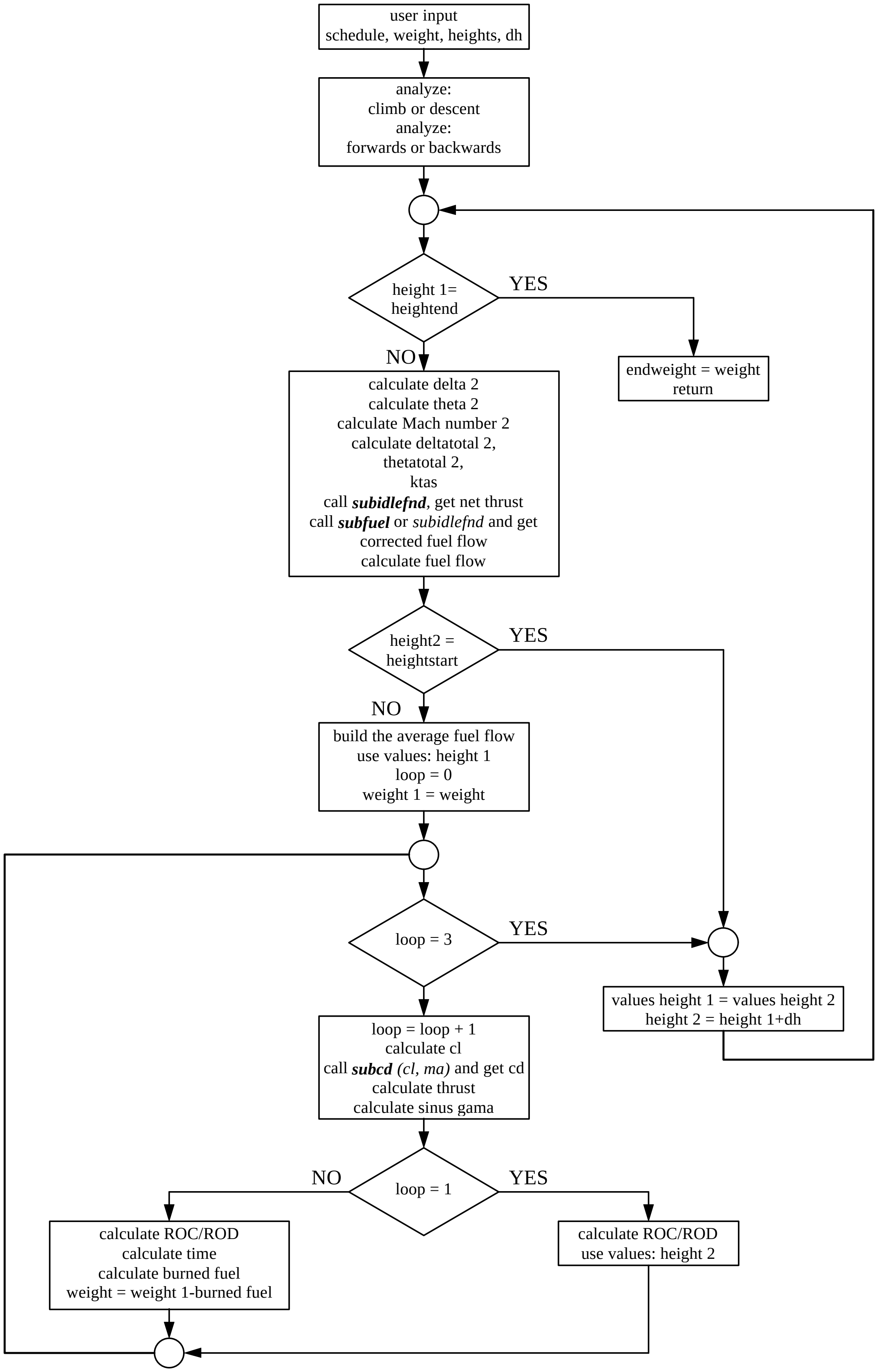


Figure 3.4 Simplified flowchart of the climb calculation

3.3 Subroutines

3.3.1 Calculation of the drag coefficient

The available drag polar is not based on an equation, but exists in table form, where the drag coefficient depends on the Mach number and on the lift coefficient. Because it is hardly the case that the current Mach number or lift coefficient are matching with those ones in the table, it is necessary to interpolate between the table values. A linear interpolation was found accurate enough and is easy to program. The name of the macro is *subcd*.The input parameters are the lift coefficient and the Mach number, output is the drag coefficient. In case it is necessary to know one single drag coefficient, the macro may be used on its own.

This macro, *subidlefnd* and *subtakeoff* are simple interpolations. They look up in a table the four nearest values of the searched one and use the equation

$$y = \frac{y_2 - y_1}{x_2 - x_1}\left(x - x_1\right) + y_1$$

for determine it.

3.3.2 Calculation of the Corrected Fuel Flow

The tables of the corrected fuel flow are used for climb and cruise conditions. Several tables exist, each of them for a different altitude. A single table is arranged after the Mach number and the net thrust over delta for one engine. Unfortunately no values exist for the wide range from 10000 feet to 35000 feet. In order to get the fuel flow in this range as accurately as possible, an iteration by Newtons method of the fourth order is used. For the whole calculation, three subroutines are needed.

The main one, called *subfuel*, defines which tables are used for the iteration. Input data comprise height, Mach number and net thrust over delta. It calls the macro *idlethrust* (next chapter), which calculates the corrected fuel flow for every required table at the specified Mach number and net thrust over delta by using a linear interpolation. After obtaining four values for four different heights these data and the specified height are passed from *subfuel* to *subnewton*, where the iteration is performed and results in the searched fuel flow.

3.3.3 Calculation of idle thrust, idle fuel flow and climb thrust

These three items can be obtained with one macro, called *subidlefnd*. All of them are available in tables depending on altitude and Mach number. The input data comprise the Mach number and the height and in addition the name of the table to be looked up, changing with the type of required value.

3.3.4 Calculation of Take-off data

The required time and fuel from brake release to a height of 1500 feet are computed by a macro called *subtakeoff*. It uses two tables, one for a normal take-off, the other for a backwards calculation from 1500 feet to brake release. Input data are the weight, either at brake release or at 1500 feet, and information about which direction is required.

3.4 Sample calculations

For every possible type of input one sample with results is given.

		INPUT	OUTPUT
Fuel on board	(lb)	70000	70000
Payload	(lb)	50000	
Brake release w.	(lb)	0	247830
Range	(nm)	0	2954
Contingency fuel	(%)	4.00	3.99

Figure 3.5 Input and results for calculation of range, given values: fuel and payload (runtime 2 min, sec)

		INPUT	OUTPUT
Fuel on board	(lb)	77422	77422
Payload	(lb)	0	49748
Brake release w.	(lb)	255000	255000
Range	(nm)	0	3276
Contingency fuel	(%)	4.00	3.99

Figure 3.6 Input and results for calculation of range, given values: fuel and brake release weight (runtime 2 min, 51 sec)

		INPUT	OUTPUT
Fuel on board	(lb)	77422	77422
Payload	(lb)	0	
Brake release w.	(lb)	0	205252
Range	(nm)	0	4105
Contingency fuel	(%)	4.00	3.99

Figure 3.7 Input and results for calculation of range, given values: fuel (no payload) (runtime 2 min, 58 sec)

		INPUT	OUTPUT
Fuel on board	(lb)	0	70570
Payload	(lb)	56600	
Brake release w.	(lb)	255000	255000
Range	(nm)	0	2870
Contingency fuel	(%)	4.00	3.99

Figure 3.8 Input and results for calculation of range, given values: payload and brake release weight (runtime 2 min, 22 sec)

		INPUT	OUTPUT
Fuel on board	(lb)	0	68775
Payload	(lb)	56600	
Brake release w.	(lb)	0	253205
Range	(nm)	2800	2799
Contingency fuel	(%)	4.00	3.95

Figure 3.9 Input and results for calculation of fuel, given values: payload and range (runtime 1 min, 50 sec)

		INPUT	OUTPUT
Fuel on board	(lb)	70000	70000
Payload	(lb)	0	34030
Brake release w.	(lb)	0	231860
Range	(nm)	3200	3200
Contingency fuel	(%)	4.00	3.99

Figure 3.10 Input and results for calculation of payload, given values: fuel and range (runtime 12 min, 57 sec)

One important application of this program is the possibility of obtaining the payload-range-diagram. (Figure 3.11)

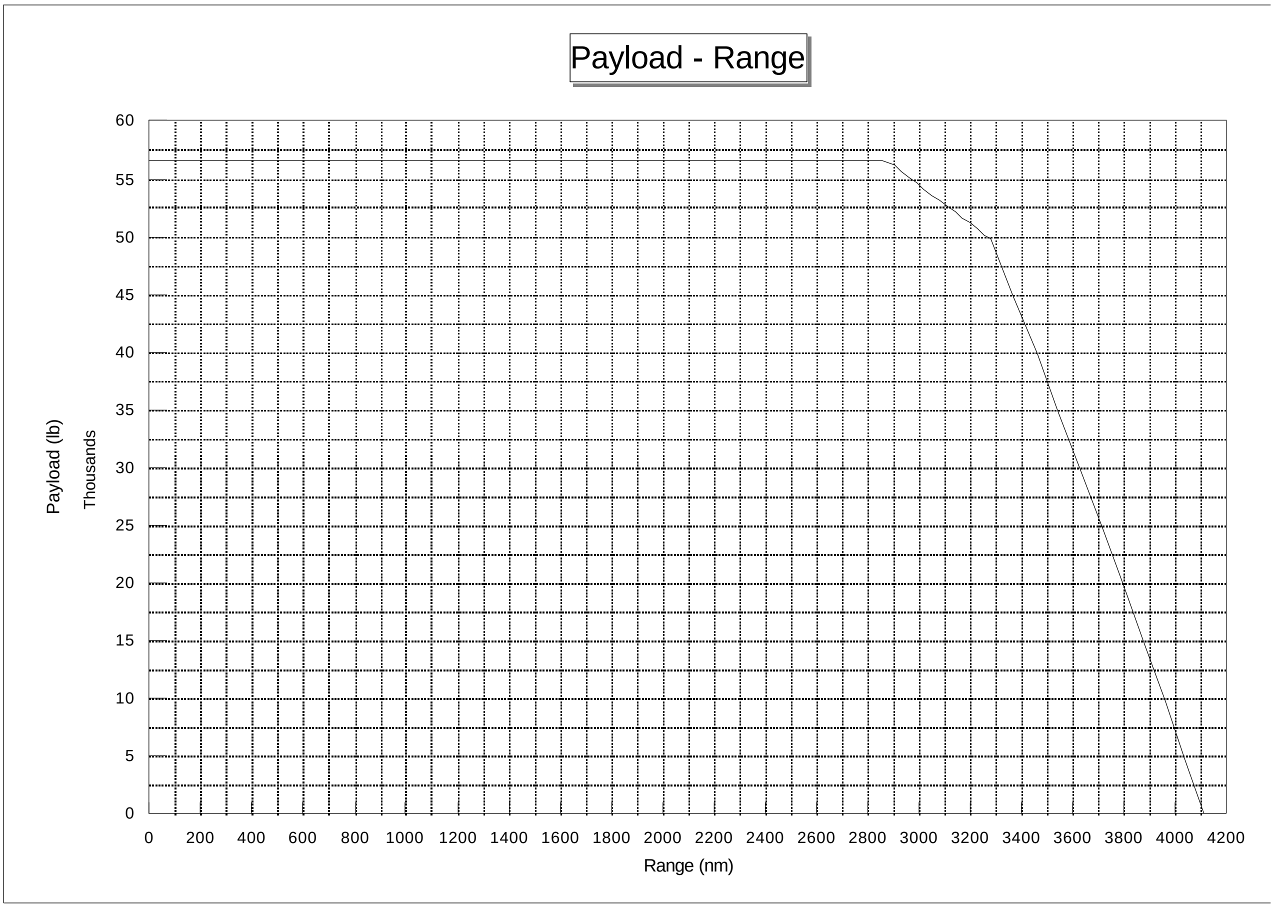


Figure 3.11 Payload Range Diagram

4 Summary

This report describes the general methods used for aircraft performance calculations on civil airliners. It starts by explaining the characteristics of the International Standard Atmosphere (ISA), which is the necessary basis for estimating and comparing aircraft and engine performance. Basic equations are derived and expressions like flight level and pressure height are explained.

The flight profile considered in this report consists of a main mission and a diversion, according to the rules of the International Civil Aviation Organisation (ICAO). Thereafter a civil airliner must carry enough fuel to be able to fly to an alternate airfield and perform a hold there, for thirty minutes, after a missed approach at its destination airport. The complete main mission consists of; taxiing to the runway, take-off and climb to 1500 feet, climb to cruise altitude according to a climb speed schedule, cruise with a certain Mach number, descent to 1500 feet (according to a schedule as well) and airport approach. The same for the diversion, in addition there is the hold performed at 1500 feet. The main parts climb/descent and cruise/hold are explained in detail and the significant equations are derived.

Considering such an aircraft mission, the burned fuel, the needed time and the flown distance depend on each other and of course on the weight of the aircraft with fixed relationships. The aim of this work was to develop a program which enables the user to ascertain unknown variables of those values after entering known ones under certain conditions specified by the user e.g cruise Mach number or climb speed schedule. This program is written as a macro on a spreadsheet in *Lotus 1-2-3*. Such a macro has many advantages compared with real programing languages. For example, the user can literally watch the macro performing, since every number existing in it is on the screen. The main disadvantage is the runtime. Because a macro cannot be compiled, it needs a long time to perform especially when containing many loops and iterations like this one.

All data used, like the drag polar or the fuel flow, were provided in the Performance Engineers Manual (PEM) for a generic twin engine jet transport aircraft and transferred into *Lotus 1-2-3*.

The program consists of a main, which controls two large subroutines for climb/descent and cruise/hold. Needed parameters like the drag coefficient or the fuel flow are calculated by several smaller subroutines. Sample calculations are given and the flowcharts are provided in the appendix.

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Appendix A

Configuration tables

Tables A.1 - A.8: the actual fuel flow is equal to $\text{corrected fuel flow} \cdot d_T \ \ q_T^{0.6363}$.

Table A.1 Corrected Fuel Flow, altitude 0 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	29585
Mach	Corrected Fuel Flow (lb/hr/engine)									
0.20	1806	3176	4486	5737	6975	8232	9541	10944	12452	13862
0.30	2033	3485	4858	6170	7467	8792	10189	11687	13345	14787
0.40	2328	3768	5148	6495	7842	9222	10665	12220	13821	15201
0.45	2443	3867	5250	6613	7978	9369	10809	12330	13863	15185
0.50	2522	3945	5338	6715	8087	9469	10868	12305	13742	14980
0.60	2729	4176	5572	6928	8245	9531	10791	12038	13286	14362

Table A.2 Corrected Fuel Flow, altitude 5000 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	32971
Mach	Corrected Fuel Flow (lb/hr/engine)										
0.20	1815	3208	4526	5767	6992	8237	9541	10950	12506	14221	16148
0.30	2057	3499	4876	6187	7489	8817	10211	11719	13353	15195	17052
0.40	2352	3774	5156	6511	7868	9253	10690	12212	13851	15559	17250
0.45	2466	3879	5263	6630	8001	9395	10835	12332	13926	15546	17151
0.50	2559	3979	5368	6737	8105	9487	10901	12356	13892	15431	16954
0.60	2767	4233	5617	6944	8238	9523	10825	12181	13581	14982	16368
0.70	2902	4355	5683	6931	8127	9308	10508	11806	13125	14445	15752

Table A.3 Corrected Fuel Flow, altitude 10000 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	32096
Mach	Corrected Fuel Flow (lb/hr/engine)										
0.30	2089	3541	4919	6226	7520	8840	10226	11725	13377	15199	16613
0.40	2377	3804	5192	6544	7898	9280	10717	12244	13871	15633	16892
0.45	2495	3911	5299	6665	8034	9426	10863	12367	13942	15621	16798
0.50	2587	4009	5401	6768	8133	9512	10925	12387	13920	15516	16631
0.55	2685	4137	5536	6890	8223	9558	10918	12328	13812	15342	16412
0.60	2787	4260	5651	6972	8254	9529	10829	12177	13620	15099	16132
0.70	2946	4393	5718	6955	8142	9315	10515	11766	13153	14543	15514

TableA.4 Corrected Fuel Flow, altitude 35000 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	33000	36000
Mach	Corrected Fuel Flow (lb/hr/engine)											
0.50	2818	4492	5970	7398	8730	10071	11459	12981	14684	16625	18865	21364
0.60	2991	4705	6169	7559	8813	10057	11330	12726	14301	16120	18240	20642
0.70	3155	4798	6170	7491	8675	9841	11025	12304	13731	15366	17260	19418
0.75	3211	4725	6037	7312	8474	9625	10792	12039	13408	14947	16700	18686
0.78	3217	4655	5926	7169	8318	9461	10619	11846	13176	14651	16309	18169
0.80	3222	4613	5857	7080	8224	9365	10521	11739	13049	14485	16084	17861
0.82	3237	4586	5805	7009	8150	9289	10445	11660	12957	14369	15925	17645
0.85	3221	4526	5723	6909	8047	9187	10344	11552	12832	14210	15710	17356
0.90	3100	4343	5515	6681	7816	8954	10103	11290	12526	13830	15217	16713

Table A.5 Corrected Fuel Flow, altitude 36089 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	33000	36000
Mach	Corrected Fuel Flow (lb/hr/engine)											
0.50	2843	4514	5997	7431	8773	10126	11526	13059	14770	16714	18959	21460
0.60	3009	4728	6202	7602	8864	10116	11397	12799	14377	16198	18327	20736
0.70	3170	4823	6206	7538	8729	9902	11091	12373	13801	15435	17339	19507
0.75	3225	4761	6082	7370	8537	9692	10862	12109	13477	15016	16785	18783
0.78	3231	4695	5975	7232	8387	9533	10694	11922	13253	14730	16406	18285
0.80	3242	4648	5894	7123	8270	9412	10570	11791	13106	14552	16173	17980
0.82	3260	4622	5845	7055	8198	9341	10500	11718	13021	14444	16023	17769
0.85	3240	4554	5753	6945	8086	9229	10388	11600	12883	14268	15785	17445
0.90	3117	4365	5544	6715	7854	8994	10148	11337	12577	13885	15284	16790

Table A.6 Corrected Fuel Flow, altitude 37000 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	33000	36000
Mach	Corrected Fuel Flow (lb/hr/engine)											
0.50	2872	4532	6015	7452	8806	10172	11586	13132	14849	16794	19023	21497
0.60	3030	4748	6226	7632	8905	10168	11459	12867	14449	16268	18383	20769
0.70	3186	4845	6234	7574	8773	9953	11149	12436	13866	15499	17392	19544
0.75	3239	4784	6115	7411	8586	9746	10920	12170	13540	15079	16841	18830
0.78	3248	4721	6012	7277	8440	9593	10758	11990	13324	14801	16471	18342
0.80	3257	4677	5935	7173	8327	9473	10634	11857	13172	14616	16235	18038
0.82	3273	4648	5882	7101	8250	9396	10558	11777	13080	14499	16076	17817
0.85	3251	4579	5787	6987	8132	9278	10438	11649	12932	14314	15828	17485
0.90	3133	4385	5566	6740	7881	9024	10179	11371	12614	13925	15326	16836

Table A.7 Corrected Fuel Flow, altitude 39000 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	33000	36000
Mach	Corrected Fuel Flow (lb/hr/engine)											
0.50	2924	4556	6031	7467	8836	10222	11659	13218	14947	16891	19093	21521
0.60	3075	4770	6244	7653	8944	10227	11539	12962	14548	16355	18434	20747
0.70	3221	4866	6258	7602	8817	10013	11224	12522	13956	15578	17441	19531
0.75	3270	4812	6154	7459	8649	9823	11010	12270	13642	15173	16905	18851
0.78	3284	4755	6058	7334	8514	9683	10864	12107	13445	14916	16560	18390
0.80	3290	4720	5995	7248	8417	9577	10749	11980	13297	14736	16335	18108
0.82	3302	4688	5938	7171	8334	9492	10663	11889	13192	14606	16162	17876
0.85	3274	4612	5834	7046	8202	9356	10524	11738	13020	14396	15891	17525
0.90	3150	4426	5621	6808	7956	9103	10259	11449	12688	13994	15385	16886

Table A.8 Corrected Fuel Flow, altitude 42000 ft

F_N/d (lb/eng.)	3000	6000	9000	12000	15000	18000	21000	24000	27000	30000	33000
Mach	Corrected Fuel Flow (lb/hr/engine)										
0.50	2947	4595	6063	7491	8844	10214	11644	13198	14940	16921	19187
0.60	3107	4799	6259	7657	8939	10217	11536	12970	14582	16435	18589
0.70	3276	4861	6240	7574	8803	10020	11254	12569	14008	15617	17447
0.75	3331	4807	6136	7433	8648	9856	11079	12366	13750	15266	16952
0.78	3342	4766	6065	7338	8542	9739	10949	12214	13560	15020	16620
0.80	3346	4727	6002	7257	8456	9651	10859	12116	13444	14870	16416
0.82	3356	4695	5945	7181	8374	9567	10774	12026	13341	14742	16247
0.85	3318	4624	5848	7063	8240	9417	10608	11839	13126	14489	15947
0.90	3190	4440	5637	6831	8003	9175	10356	11563	12805	14094	15445

Table A.9 Maximum climb thrust*

Mach	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
Height (ft)	Net thrust /delta per engine (lb)														
0	**25233**	24152	**23071**	22157	**21243**	**20313**	**19671**	19074	18477						
1000	25724	24626	23528	22603	21678	20772	20112	19507	18902						
1500	25969	24863	23757	22827	21896	21002	20333	19724	19115						
2000	26215	25100	23986	23050	22114	21231	20553	19941	19328						
3000	26705	25574	24443	23496	22549	21691	20995	20374	19753						
4000	27196	26048	24901	23943	22985	22150	21436	20807	20179						
5000	**27687**	26523	**25358**	24389	**23420**	**22609**	**21877**	21241	**20604**	20091	**19577**				
6000			25830	24865	23899	23087	22353	21704	21073	20548	20024				
7000			26302	25340	24378	23566	22829	22168	21542	21006	20471				
8000			26773	25816	24858	24044	23306	22631	22010	21464	20917				
9000			27245	26291	25337	24523	23782	23095	22479	21922	21364				
10000			**27717**	26767	**25816**	**25001**	**24258**	**23558**	**22948**	22380	**21811**				
11000			28219	27269	26318	25507	24760	24067	23455	22892	22330				
12000			28722	27771	26820	26013	25262	24577	23961	23405	22848				
13000			29224	28273	27323	26518	25764	25086	24468	23917	23367				
14000			29727	28776	27825	27024	26266	25596	24974	24430	23885				
15000			**30229**	29278	**28327**	**27530**	**26768**	**26105**	**25481**	24943	**24404**	23932	**23459**		
16000			30655	29727	28799	28022	27272	26623	26017	25489	24957	24485	24013		
17000			31081	30175	29270	28513	27777	27141	26553	26036	25510	25039	24567		
18000			31506	30624	29742	29005	28281	27658	27089	26582	26064	25592	25120		
19000			31932	31073	30213	29496	28786	28176	27625	27129	26617	26146	25674		
20000			**32358**	31522	**30685**	29988	**29290**	**28694**	**28161**	**27675**	**27170**	26699	**26228**	25883	**25537**
21000					31028	30358	29689	29120	28623	28163	27699	27236	26772	26435	26098
22000					31371	30729	30088	29546	29086	28651	28228	27773	27316	26988	26659
23000					31713	31100	30487	29972	29548	29138	28756	28309	27859	27540	27221
24000					32056	31471	30886	30398	30011	29626	29285	28846	28403	28093	27782
25000					**32399**	31842	**31285**	**30824**	**30473**	**30114**	**29814**	**29383**	**28947**	28645	**28343**
26000							31643	31238	30925	30617	30371	30030	29676	29409	29125
27000							32001	31652	31376	31120	30927	30678	30404	30172	29907
28000							32360	32066	31828	31623	31484	31325	31133	30936	30690
29000							32718	32480	32279	32125	32040	31972	31861	31700	31472
30000							33076	32894	32731	32628	32597	32620	32590	32463	32254
31000							**33434**	33308	**33182**	**33131**	**33153**	**33267**	**33318**	**33227**	**33036**
32000							33681	33587	33493	33506	33573	33765	33886	33892	33862
33000							33928	33866	33804	33880	33993	34263	34454	34558	34688
34000							34174	34144	34114	34255	34413	34760	35022	35223	35513
35000							**34421**	34423	**34425**	34629	**34833**	**35258**	**35590**	**35888**	**36339**
36089							**34529**	34547	**34564**	34793	**35021**	**35455**	**35933**	**36240**	**36743**
37000							**34426**	34441	**34456**	34683	**34909**	**35339**	**35812**	**36110**	**36602**
38000							34315	34327	34339	34563	34787	35212	35681	35969	36449
39000							**34203**	34213	**34222**	34443	**34664**	**35085**	**35549**	**35828**	**36296**
40000							34061	34062	34062	34302	34541	34957	35417	35685	36137
41000							33920	33911	33903	34160	34418	34830	35284	35543	35978
42000							**33778**	33761	**33743**	34019	**34295**	**34702**	**35152**	**35400**	**35819**

(* Numbers which are not bold were obtained by linear interpolation)

Table A.10 Minimumidle inflight thrust*

Mach	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
Height (ft)	Net thrust/delta per engine (lb)														
0	792	658	523	412	301	195	72	-79	-229						
5000	903	757	611	500	388	276	146	-5	-155	-333	-510				
10000			703	591	478	363	232	87	-77	-257	-437				
15000			816	696	575	453	318	168	1	-179	-359	-550	-740		
20000			946	819	691	555	419	263	91	-89	-278	-472	-665	-886	-1107
25000					803	666	528	366	188	0	-188	-377	-573	-792	-1010
31000							828	644	460	274	88	-88	-267	-472	-676
35000							1504	1290	1075	869	662	463	267	29	-144
36089							1740	1508	1275	1063	850	644	443	196	26
37000							1927	1693	1459	1239	1019	809	598	350	177
39000							2404	2147	1889	1653	1416	1194	973	715	550
41000							2829	2550	2271	2020	1770	1534	1295	1036	873
42000							3253	2953	2652	2388	2123	1873	1617	1356	1195

(*Numbers which are not bold were obtained by linear interpolation)

Table A.11 : the actual fuel flow is equal to $\text{corrected fuel flow} + 505 \cdot d_T \; q_T^{0.6363}$.

Table A.11 Minimum idle fuel flow*

Mach	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
Height (ft)	Minimum corrected idle fuel flow per engine (lb/hr/eng.)														
0	871	863	855	843	830	815	791	754	716						
5000	810	803	796	787	778	766	744	718	691	657	623				
10000			729	726	723	715	701	683	661	633	605				
15000			661	660	658	653	644	632	617	598	579	553	526		
20000			616	613	610	603	595	583	569	554	539	520	500	476	452
25000					564	559	554	545	534	520	506	490	471	453	434
31000							529	522	515	507	498	488	477	465	451
35000							557	552	547	540	533	524	515	505	494
36089							564	559	554	548	541	533	525	515	504
37000							569	564	559	553	547	540	531	522	512
39000							580	575	570	565	559	552	545	537	527
41000							587	582	578	572	567	561	554	547	538
42000							593	589	585	580	575	569	563	556	548

(*Numbers which are not bold were obtained by linear interpolation)

Table A.12 Time and fuel from brake release to 1500 ft, final speed 250 KCAS

Weight/delta (lb)	Fuel (lb)	Time (min)
160000	590	1.578
180000	675	1.722
200000	760	1.86
220000	845	2.004
240000	930	2.148

Table A.13 Time and fuel from 1500 ft to brake release (for backwards calculation)

Weight/delta (lb)	Fuel (lb)	Time (min)
159410	590	1.578
179325	675	1.722
199240	760	1.86
219155	845	2.004
239070	930	2.148

Table A.14 Recommended holding speed at 1500 ft

Weight/delta (lb)	Mach
100000	0.260
150000	0.319
200000	0.368
250000	0.412
300000	0.451
350000	0.487
400000	0.521
450000	0.553
500000	0.582
550000	0.611
600000	0.638
650000	0.664
700000	0.689
750000	0.713
800000	0.737
850000	0.759
851400	0.760
1500000	0.760

Table A.15 High speed drag polare

	Lift coefficient								
	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550
Mach	Drag coefficient								
0.30	0.01690	0.01786	0.01905	0.02046	0.02205	0.02384	0.02593	0.02838	0.03108
0.35	0.01680	0.01774	0.01889	0.02025	0.02181	0.02359	0.02566	0.02808	0.03076
0.40	0.01671	0.01761	0.01872	0.02005	0.02158	0.02334	0.02539	0.02778	0.03044
0.45	0.01663	0.01752	0.01861	0.01991	0.02140	0.02312	0.02512	0.02749	0.03014
0.50	0.01655	0.01741	0.01849	0.01977	0.02125	0.02293	0.02488	0.02718	0.02980
0.55	0.01650	0.01737	0.01844	0.01972	0.02117	0.02283	0.02475	0.02700	0.02959
0.60	0.01662	0.01747	0.01851	0.01975	0.02122	0.02290	0.02479	0.02705	0.02966
0.65	0.01684	0.01766	0.01867	0.01989	0.02138	0.02307	0.02498	0.02722	0.02995
0.70	0.01713	0.01789	0.01887	0.02009	0.02159	0.02330	0.02523	0.02750	0.03035
0.71	0.01720	0.01795	0.01892	0.02013	0.02163	0.02336	0.02530	0.02759	0.03045
0.72	0.01726	0.01800	0.01896	0.02017	0.02167	0.02341	0.02536	0.02768	0.03055
0.73	0.01732	0.01805	0.01902	0.02023	0.02173	0.02348	0.02543	0.02779	0.03067
0.74	0.01739	0.01811	0.01908	0.02031	0.02181	0.02355	0.02552	0.02790	0.03079
0.75	0.01746	0.01817	0.01915	0.02039	0.02190	0.02363	0.02562	0.02804	0.03094
0.76	0.01753	0.01824	0.01923	0.02048	0.02199	0.02373	0.02575	0.02820	0.03113
0.77	0.01760	0.01832	0.01931	0.02057	0.02210	0.02384	0.02588	0.02838	0.03137
0.78	0.01769	0.01843	0.01943	0.02069	0.02221	0.02395	0.02604	0.02859	0.03168
0.79	0.01786	0.01860	0.01961	0.02087	0.02235	0.02408	0.02625	0.02887	0.03212
0.80	0.01813	0.01890	0.01991	0.02115	0.02258	0.02430	0.02659	0.02938	0.03293
0.81	0.01854	0.01934	0.02034	0.02154	0.02297	0.02473	0.02723	0.03038	0.03456
0.82	0.01918	0.01997	0.02097	0.02218	0.02366	0.02559	0.02850	0.03234	0.03741
0.83	0.02020	0.02095	0.02192	0.02315	0.02481	0.02720	0.03076	0.03545	0.04140
0.84	0.02165	0.02236	0.02331	0.02456	0.02652	0.02964	0.03400	0.03955	0.04619
0.85	0.02355	0.02422	0.02520	0.02657	0.02887	0.03267	0.03775	0.04389	0.05100
0.86	0.02590	0.02652	0.02760	0.02921	0.03187	0.03625	0.04194	0.04839	0.05583
0.87	0.02870	0.02926	0.03052	0.03250	0.03552	0.04037	0.04640	0.05315	0.06066

Table A.16 High speed drag polare

	Lift coefficient								
	0.600	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
Mach	Drag coefficient								
0.30	0.03393	0.03697	0.04030	0.04386	0.04763	0.05164	0.05595	0.06051	0.06526
0.35	0.03361	0.03665	0.03995	0.04350	0.04730	0.05135	0.05566	0.06021	0.06496
0.40	0.03331	0.03638	0.03964	0.04318	0.04702	0.05111	0.05537	0.05990	0.06482
0.45	0.03302	0.03609	0.03940	0.04288	0.04677	0.05089	0.05523	0.05988	0.06496
0.50	0.03272	0.03583	0.03913	0.04263	0.04655	0.05072	0.05516	0.05996	
0.55	0.03250	0.03565	0.03900	0.04259	0.04649	0.05070	0.05523	0.06006	
0.60	0.03266	0.03591	0.03940	0.04320	0.04725	0.05153	0.05608		
0.65	0.03308	0.03651	0.04026	0.04430	0.04858	0.05310			
0.70	0.03358	0.03732	0.04145	0.04587	0.05058				
0.71	0.03369	0.03750	0.04173	0.04624	0.05104				
0.72	0.03380	0.03767	0.04200	0.04660	0.05150				
0.73	0.03394	0.03785	0.04225	0.04700	0.05203				
0.74	0.03409	0.03804	0.04258	0.04743					
0.75	0.03427	0.03829	0.04296	0.04790					
0.76	0.03452	0.03864	0.04332	0.04850					
0.77	0.03487	0.03923	0.04412	0.04944					
0.78	0.03541	0.04016	0.04556	0.05147					
0.79	0.03618	0.04161	0.04790						
0.80	0.03753	0.04388	0.05125						
0.81	0.04008	0.04733	0.05550						
0.82	0.04390	0.05176	0.06029						
0.83	0.04854	0.05666	0.06532						
0.84	0.05362	0.06174	0.07050						
0.85	0.05875	0.06698	0.07590						
0.86	0.06390	0.07239	0.08157						
0.87	0.06908	0.07796	0.08750						

Appendix B

Airspeed conversions

Calibrated airspeed to Mach number:

$$M = \sqrt{5\left[\left(\frac{1}{d}\left\{\left[1+0.2\left(\frac{v_C}{661.4786}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]}$$

Equivalent airspeed to Mach number:

$$M = \frac{v_E}{661.4786}\sqrt{\frac{1}{d}}$$

True airspeed to Mach number:

$$M = \frac{v}{661.4786\sqrt{q}}$$

Mach number to Calibrated air speed:

$$v_C = 1479.1\sqrt{\left[\left(d\left[\left(0.2\ M^2+1\right)^{3.5}-1\right]+1\right)^{\frac{1}{3.5}}-1\right]}$$

Equivalent airspeed to calibrated airspeed:

$$v_C = 1479.1\sqrt{\left[\left(d\left\{\left[1+\frac{1}{d}\left(\frac{v_E}{1479.1}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]}$$

True airspeed to calibrated airspeed:

$$v_C = 1479.1\sqrt{\left[\left(d\left\{\left[1+\frac{1}{q}\left(\frac{v}{1479.1}\right)^2\right]^{3.5}-1\right\}+1\right)^{\frac{1}{3.5}}-1\right]}$$

Mach number to equivalent airspeed:

$$v_E \;=\; 661.4786\, M\, \sqrt{d}$$

Calibrated airspeed to equivalent airspeed:

$$v_E \;=\; 1479.1 \sqrt{d \left[\left(\frac{1}{d} \left\{ \left[1 + 0.2 \left(\frac{v_C}{661.4786} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right)^{\frac{1}{3.5}} - 1 \right]}$$

True airspeed to equivalent airspeed:

$$v_E \;=\; v \sqrt{s} \;=\; v \sqrt{\frac{d}{s}}$$

Mach number to true airspeed:

$$v \;=\; 661.4786\, M\, \sqrt{q}$$

Calibrated airspeed to true airspeed:

$$v \;=\; 1479.1 \sqrt{q \left[\left(\frac{1}{d} \left\{ \left[1 + 0.2 \left(\frac{v_C}{661.4786} \right)^2 \right]^{3.5} - 1 \right\} + 1 \right)^{\frac{1}{3.5}} - 1 \right]}$$

Equivalent to true airspeed

$$v \;=\; \frac{v_E}{\sqrt{s}} \;=\; v_E \sqrt{\frac{q}{d}}$$

Appendix C

Flowcharts

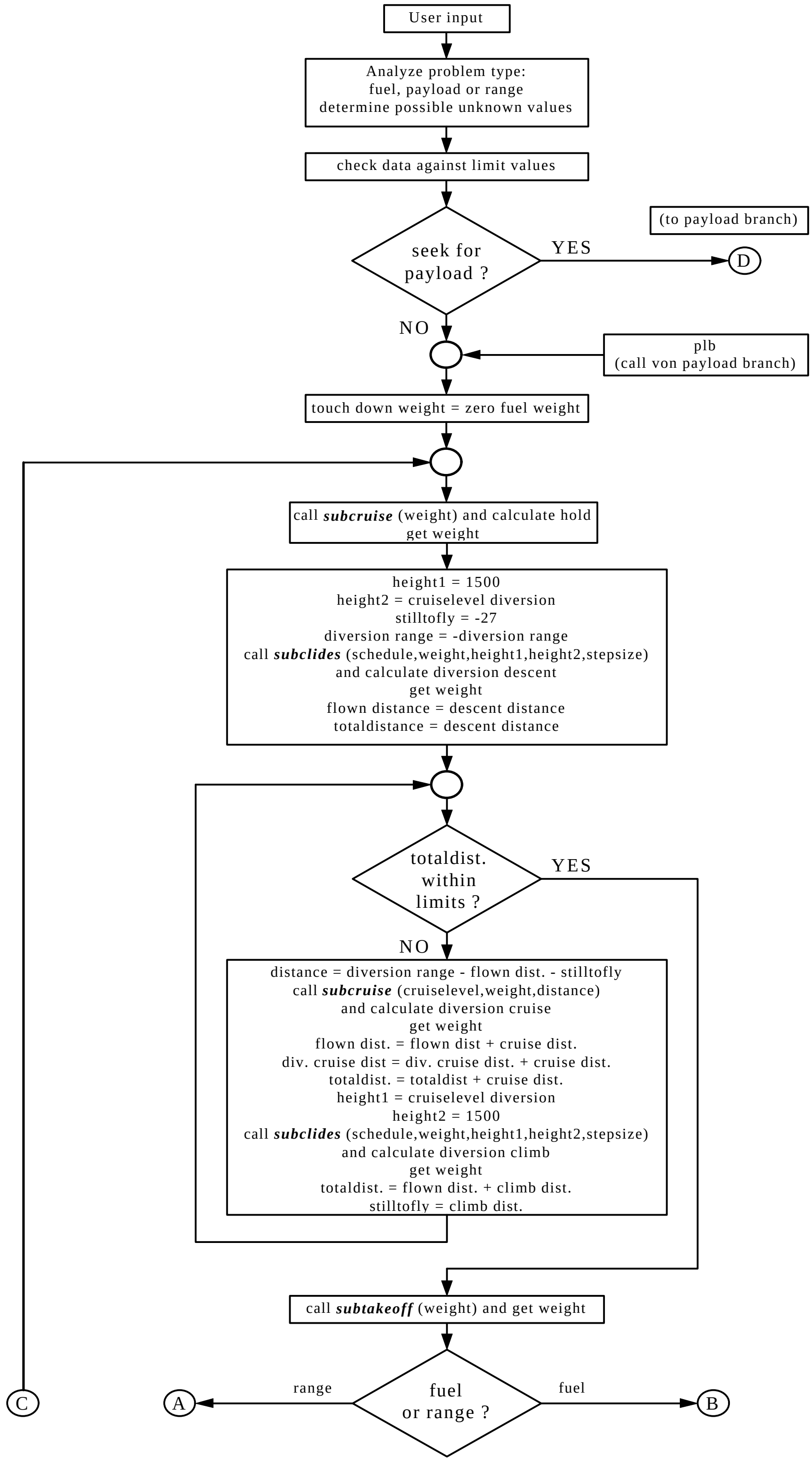

Figure C.1 Flowchart of main macro (Diversion for fuel and range calculation)

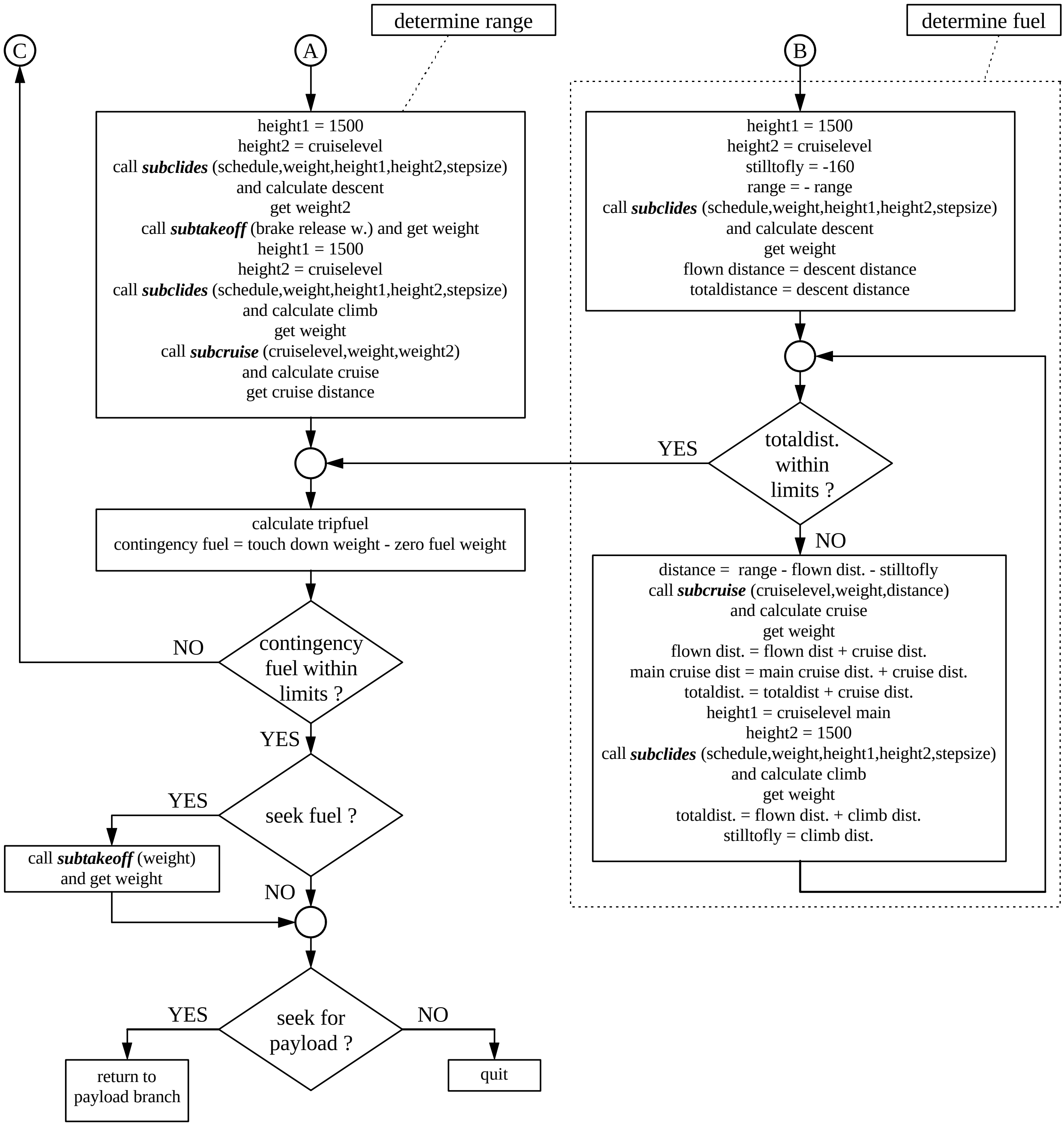

Figure C.2 Flowchart of main macro (Fuel and range calculation)

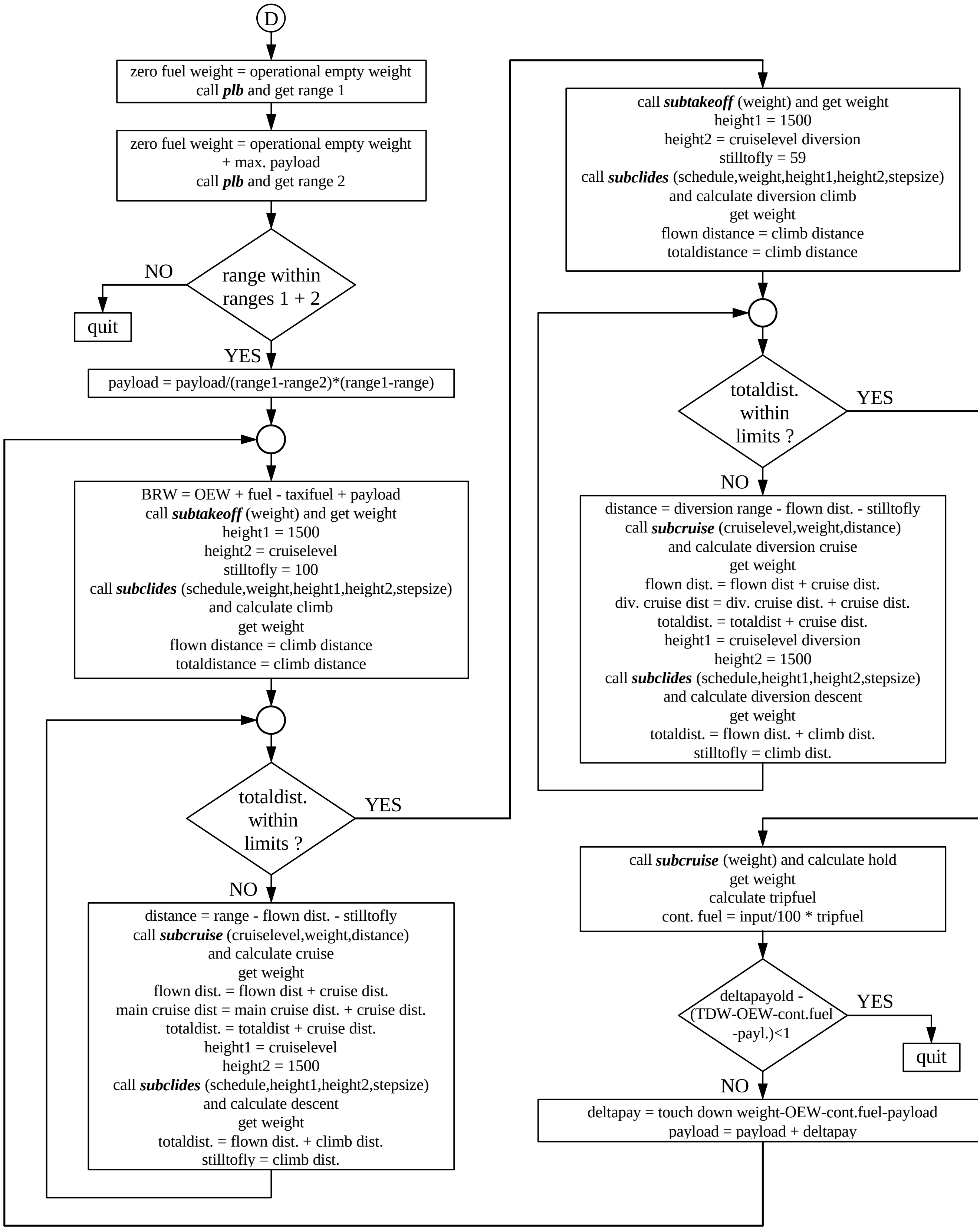

Figure C.3 Flowchart of main macro (Calculation of payload)

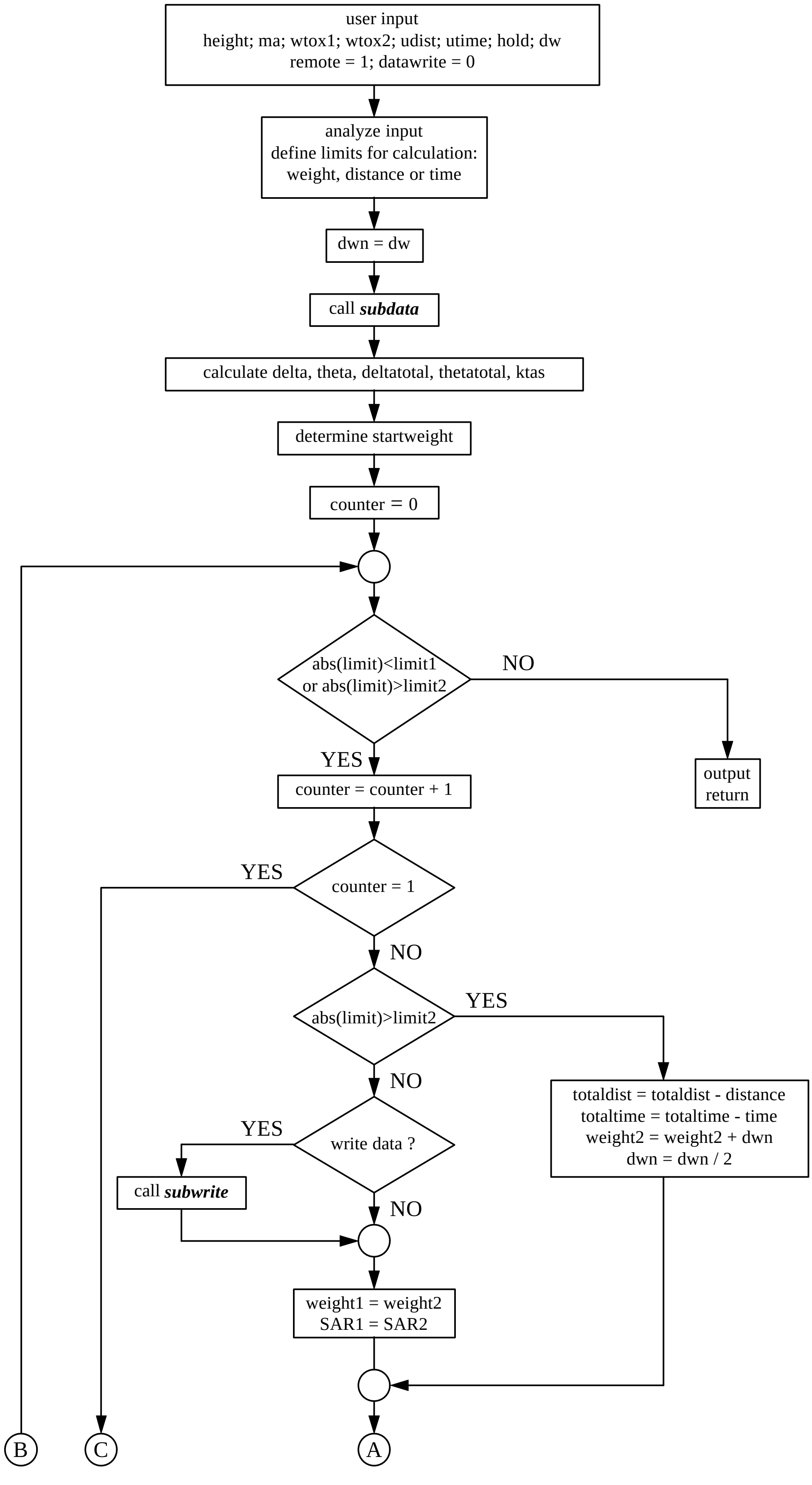

Figure C.4 Flowchart of macro for cruise calculation *subcruise*

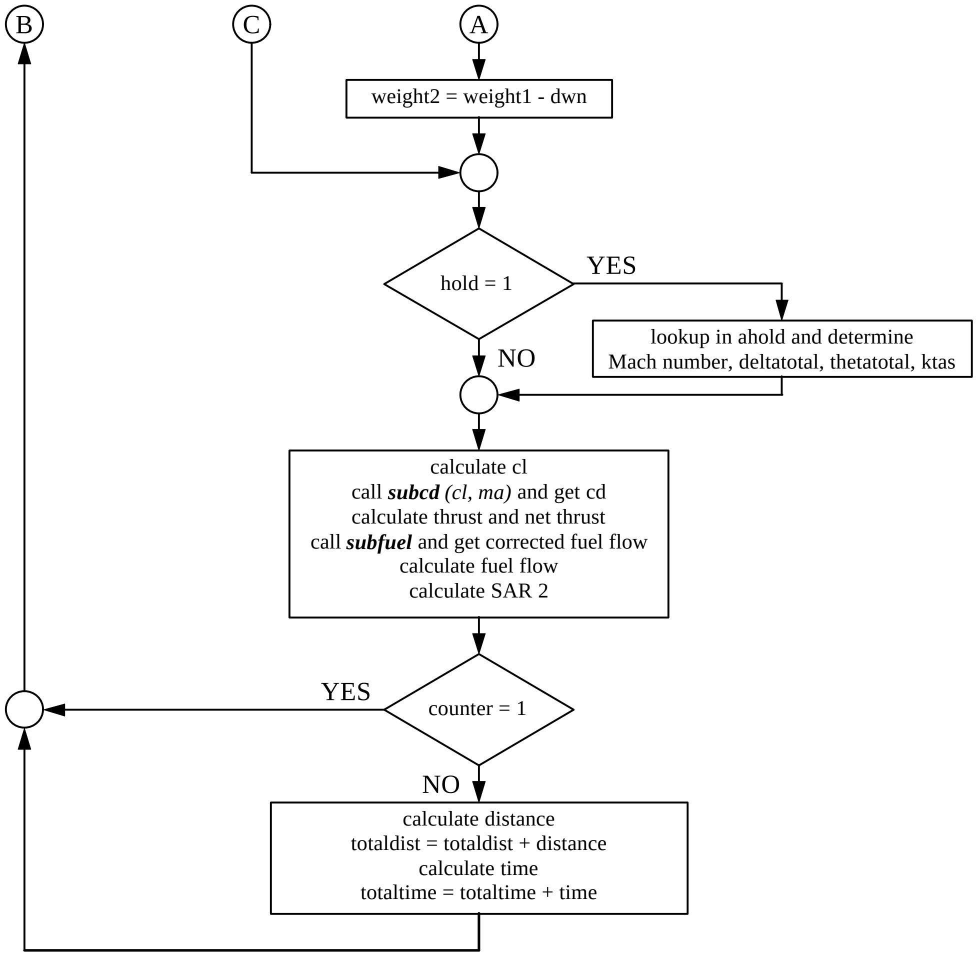

Figure C.5 Flowchart of macro for cruise calculation *subcruise*

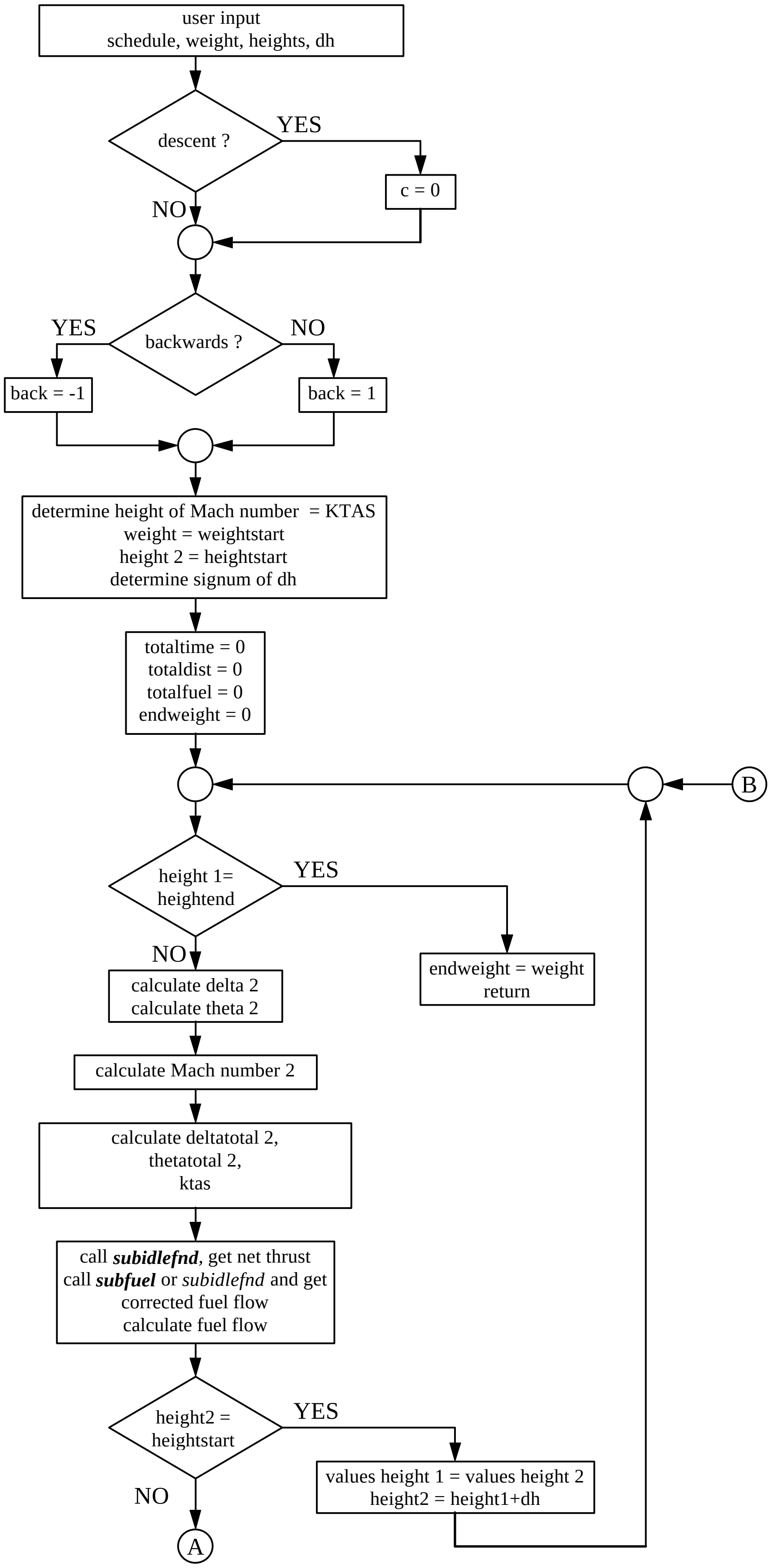

Figure C.6 Flowchart of macro for climb and descent *subclides*

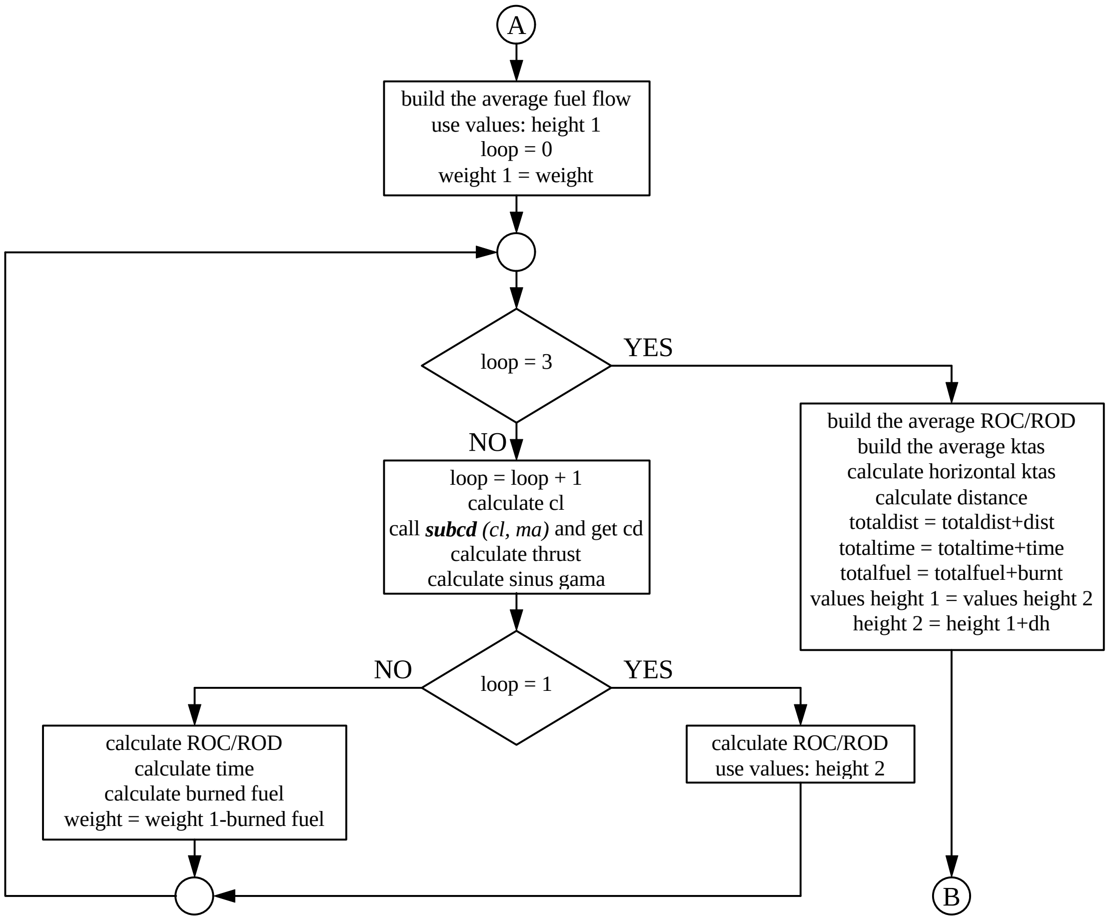

Figure C.7 Flowchart of macro for climb and descent *subclides*

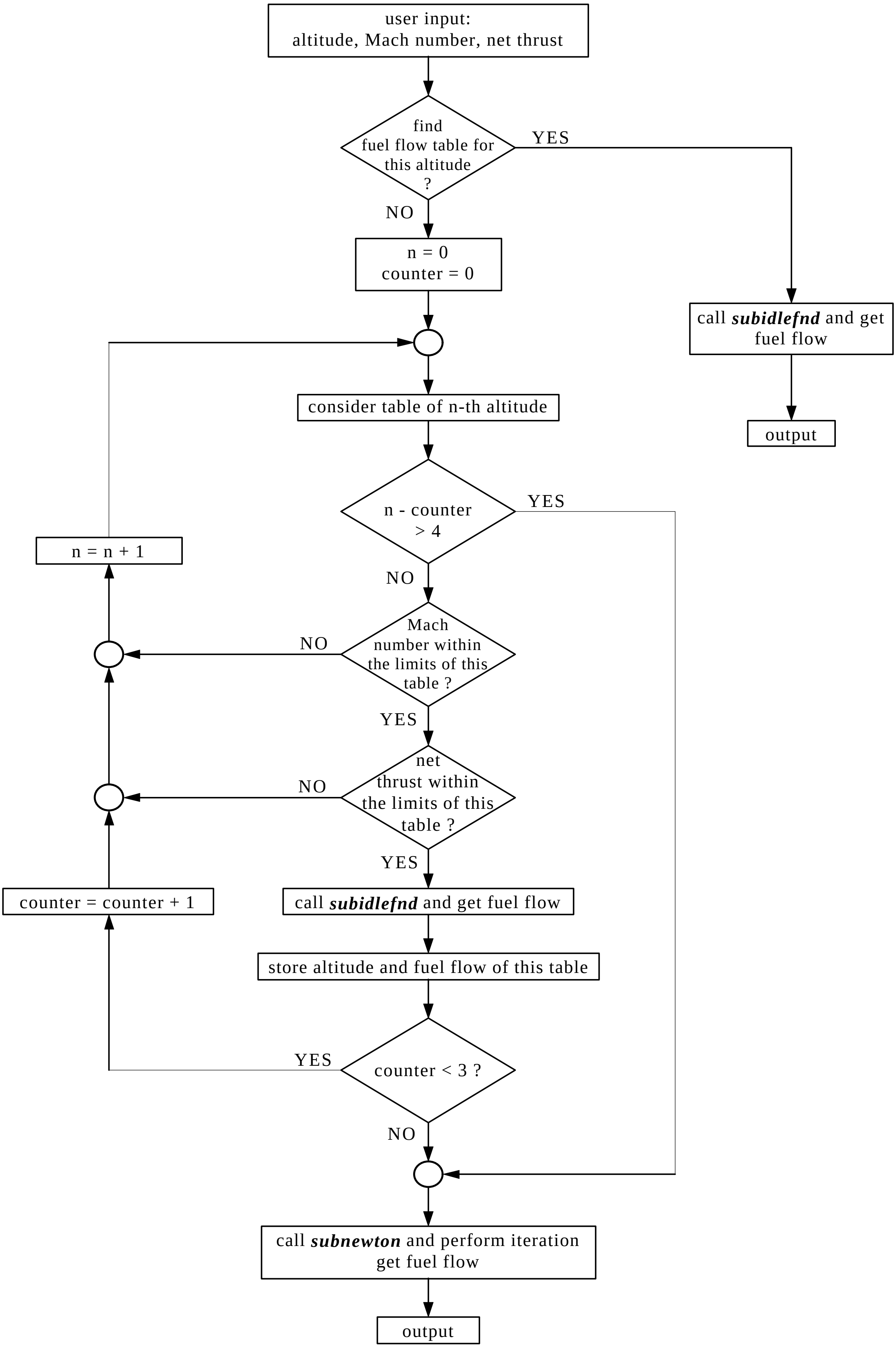

Figure C.8 Flowchart of *subfuel* (controls the interpolation of fuel flow)

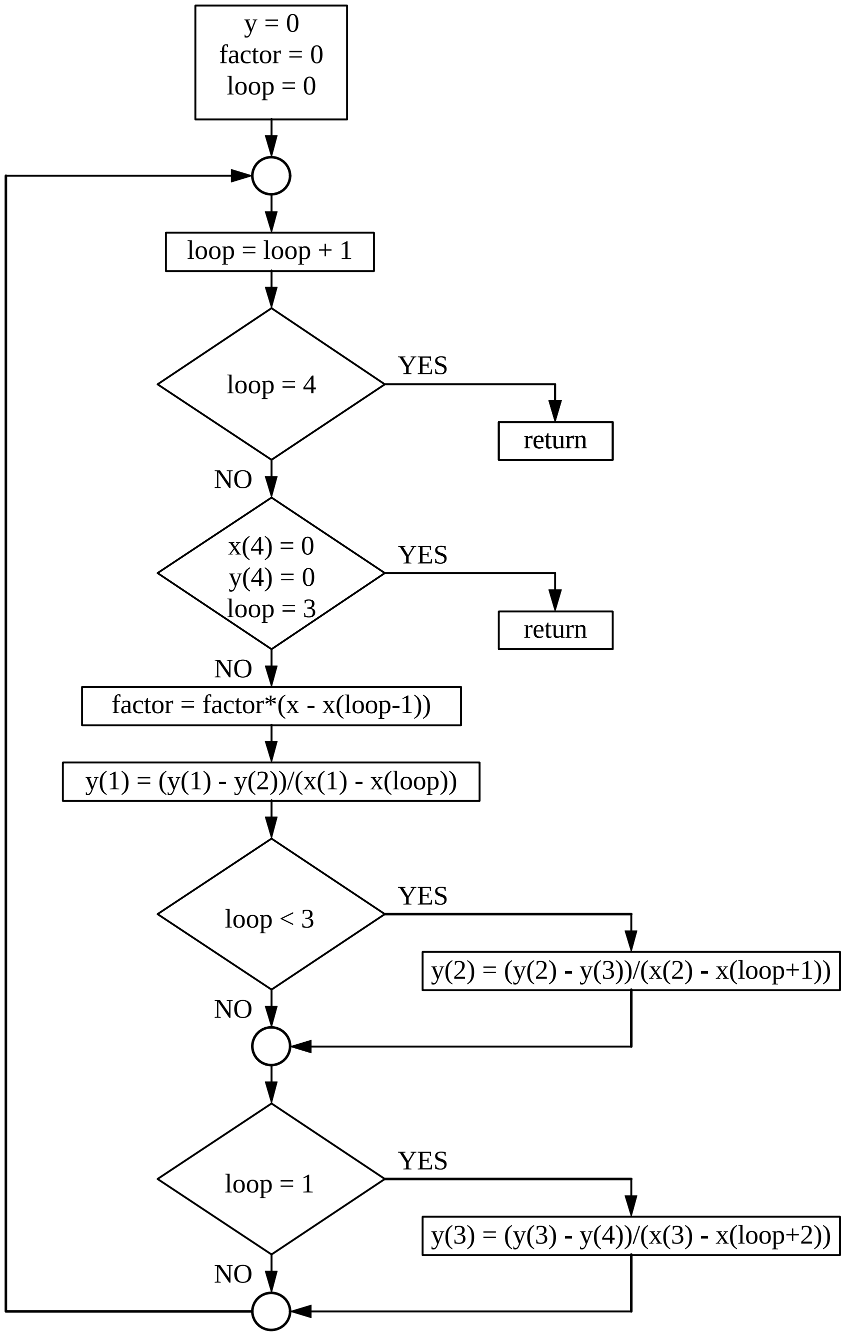

Figure C.9 Flowchart of Newtons interpolation *subnewton*

Flowcharts for the macros *subidlefnd*, *subcd* and *subtakeoff* are not provided, since they consists of simple linear interpolations. (Chapter 3.3.1)