

# Comparing Modes of Transportation with an Improved Kármán-Gabrielli Diagram

Task for a *Bachelor Thesis*

## Background

Start of the considerations is:

GABRIELLI, Giuseppe, VON KARMAN, Theodore, 1950: What price speed? Specific power required for propulsion of vehicles. *Mechanical Engineering*, vol. 72 (1950), no. 10, pp. 775-781. Available from: <https://perma.cc/5FZH-YGTR>.

Three parameters are collected for each mode of transportation: total weight ( $W = mg$ ), max. power ( $P$ ) and max. speed ( $V$ ).  $P/(W V)$  is called "specific power". The idea was revisited in 2005 (<https://perma.cc/43XQ-BJRW>). Now  $P/(W V)$  is called "specific resistance". The concept is discussed further e.g. on [Wikipedia](#). Critique of the "Kármán-Gabrielli Diagram" (KG Diagram):

- Only the percentage of power should be considered which is used in cruise.
- Cruise speed should be used not maximum speed.
- Payload should be used instead of total mass.
- Instead of shaft power and speed, energy (fuel) consumption should be used, or even better, primary energy consumption.

Another change to the original publication from 1950: The inverse is used in the KG Diagram:  $(W V)/P = W/D = L/D$ , with lift ( $L$ ) and drag ( $D$ ) we get the glide ratio (L over D), well known in aviation.  $L/D$  or  $W/D$  is an efficiency to carry weight. In this new way, the KG Diagram may be plotted like [this](#).

The plot shows a straight line in the log-log-plot from top left to bottom right. In a normal plot this line is a hyperbola, which resembles a [Pareto front](#). High speeds result in reduced efficiency. Apparently, you cannot have both. A straight line in the log-log-plot can be drawn through each point, characterizing a vehicle.

Every  $L/D$  can be calculated from  $L/D = a_{L/D} / V$ . In this equation  $a_{L/D}$  is a new performance parameter  $a_{L/D} = L/D V$  (in m/s) that combines the benefits of efficiency and speed. Every vehicle can be assigned such new "transport figure of merit". Two vehicles on the same line have the same "transport figure of merit". The underlying (philosophical) assumption is: The "transport figure of merit" is the same, if we (e.g.) double speed, while efficiency is halved.

Note also that Specific Air Range,  $SAR = L/D V / (c g) \cdot 1/m$  is the inverse of fuel consumption for jet aircraft ( $c$  is the thrust-specific fuel consumption,  $m$  is the aircraft's mass).  $L/D V / (c g)$

has the units of meter (m). It is called Breguet factor and is the range of an aircraft that has (unrealistic) 63.2% of its mass at take-off assigned to fuel.

Moreover,  $P/(W V)$  with  $V = s/t$  can also be understood as a proxy of  $(P t)/(s W)$ , which can be understood as energy (or fuel) consumption (per distance and) per weight. We plot the inverse of this energy consumption,  $E$  per distance and payload. In this way the diagram looks the same as before with  $L/D$ . We know, one form of energy (e.g. electricity) cost more than another form of energy (e.g. kerosene). Basically, we pay not for energy, but for "useful" energy, which can be described by [exergy](#). Alternatively, we calculate [primary energy](#) from the vehicle's energy requirement. In this case, the "transport figure of merit" follows from  $(s m_{PL})/E = a_E / V$ . In this equation  $a_E = (s m_{PL})/E \cdot V$  is another form of the "transport figure of merit" now expressed in second (s). This converts speed in m/s into the inverse of energy per distance and payload, given in mkg/J. Investigation could go from energy to emissions or alternatively to operating costs. Speed and energy consumption could be based on their maximum values in the diagram to eliminate scaling effects. In this way, it shows a non-preferenced "transport figure of merit". If weighting factors  $k$  for relative  $(s m_{PL})/E$  and  $1-k$  for relative  $V$  are introduced to express preference for high speed or low energy consumption ( $0 \leq k \leq 1$ ), an a priori linear scaling to express preference is introduced.

As speed increases, so does primary energy. This means, we have to pay for the energy to reach the desired speed. This is the answer to the original question: "What price speed?". We find, some modes of transportation are faster, some consume less energy and some have a higher "transport figure of merit". It is a bicriteria optimization more generally treated as [multi-objective optimization](#).

## Task

Illustrate what is explained in the Background. Using the "transport figure of merit", which modes of transportation are better in general? Which in aviation? These are the subtasks:

- Recall systematically the development and reception in literature of the Kármán-Gabrielli Diagram. Look at its various performance parameters (specific power, specific resistance, efficiency to carry weight, energy consumption, primary energy consumption) and related fundamental equations.
- Consider different modes of transportation: On the road (walking, biking, horse, car, truck, ...), on tracks, in the air, on water, and in [pipelines](#). In aviation, consider at best all [classes](#) under "Aeronautics/Aircraft" (fixed wing aircraft – divided by cruise speed and propulsion principle, rotorcraft, lighter than air, unpowered flight). For each mode, show required equations and considerations to determine power, drag, vehicle and payload mass, energy (fuel) consumption, and primary energy consumption.
- Collect values for all required parameters for each mode of transportation. Show and discuss possible sources of information.
- Plot all mentioned KG Diagrams and calculate the "transport merit" in various forms for all modes of transportation.
- Discuss the results.

The report has to be written in English based on German or international standards on report writing.