Hochschule für Angewandte Wissenschaften Hamburg

Hamburg University of Applied Sciences

AERO – AIRCRAFT DESIGN AND SYSTEMS GROUP

ESTIMATING THE OSWALD FACTOR FROM BASIC AIRCRAFT GEOMETRICAL PARAMETERS

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Deutscher Luft- und Raumfahrtkongress 2012

German Aerospace Congress 2012

Berlin, Germany, 10.-12.09.2012 **DLRK 2012**

Introduction

Airplane drag

zero-lift drag

drag due to lift

$$C_D = C_{D,0} + C_{D,i} = C_{D,0} + \frac{{C_L}^2}{\pi A e} = C_{D,0} + \frac{{C_L}^2}{\pi A}(1+\delta)$$

Oswald factor *e*

Method 1:

Oswald Factor *e* Calculated without Input of C_{Do}

Application: Preliminary Sizing

$$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M}$$

Method 1: Calculate Oswald Factor *e* without Input of C_{Do}

$$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M}$$

e	Oswald factor:
	correction factor for the aspect ratio to calculate drag due to lift
e_{theo}	theoretical Oswald factor, invisid drag due to lift only
$k_{e,F}$	correction factor: losses due to the fuselage
k_{e,D_0}	correction factor: viscous drag due to lift
$k_{e,M}$	correction factor: compressibility effects on induced drag

Method 1: Oswald Factor *e* without Input of C_{Do}

$$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M}$$

Aircraft category	d_F / b	$k_{e,F}$	k_{e,D_0}
All	0.114	0.974	-
Jet	0.116	0.973	0.873
Business Jet	0.120	0.971	0.864
Turboprop	0.102	0.979	0.804
General Aviation	0.119	0.971	0.804

e_{theo} see next pages

$$k_{e,F} = 1 - 2\left(\frac{d_F}{b}\right)^2$$

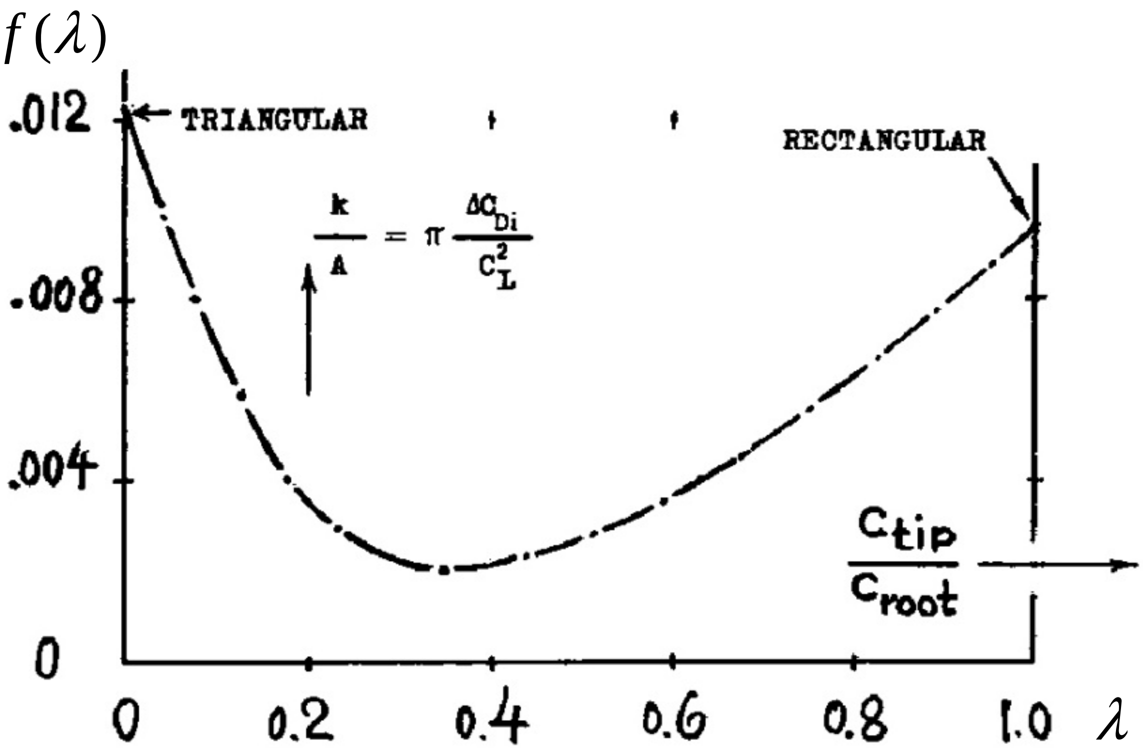
$$k_{e,M} = a_e\left(\frac{M}{M_{comp}} - 1\right)^{b_e} + c_e$$

$a_e < 0; \quad c_e = 1$

$$a_e = -0.00152$$
$$b_e = 10.82$$
$$c_e = 1$$
$$M_{comp} = 0.3$$

Literature Study: HÖRNER

Corrections to induced drag for unswept wings as a function of taper ratio λ



$$f(\lambda) = k / A$$

$$C_{Di} = (1 + k)\, C_L^2 / \pi A$$

$$k = \delta$$

$$e_{theo} = \frac{1}{1+\delta}$$

$$e_{theo} = \frac{1}{1 + f(\lambda) \cdot A}$$

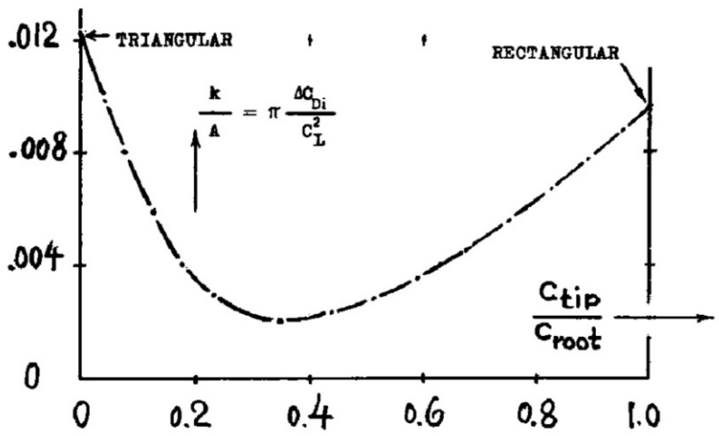
Method 1: e_{theo} for unswept Wings

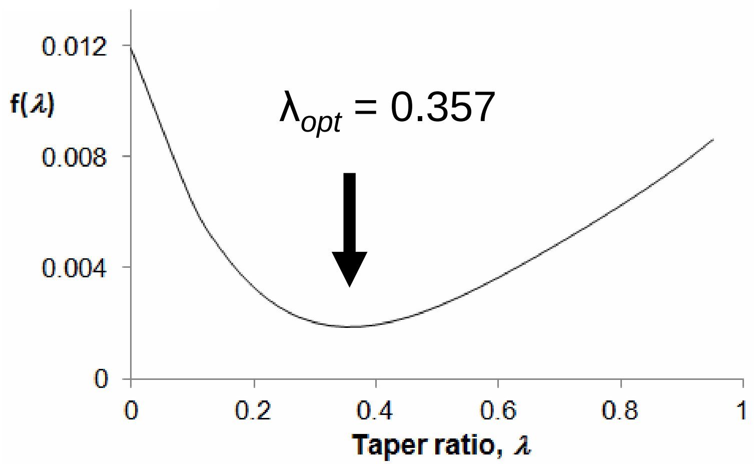
Estimating a Theoretical Oswald Factor

- Approximation of Hörner's function:

$$f(\lambda) = 0.0524\,\lambda^4 - 0.15\lambda^3 + 0.1659\lambda^2 - 0.0706\lambda + 0.0119$$

From the derivative of the function f(λ), the optimum taper ratio for unswept wings is $\lambda_{opt} = 0.357$





Literature Study: NACA Report 921

648 REPORT NO. 921—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

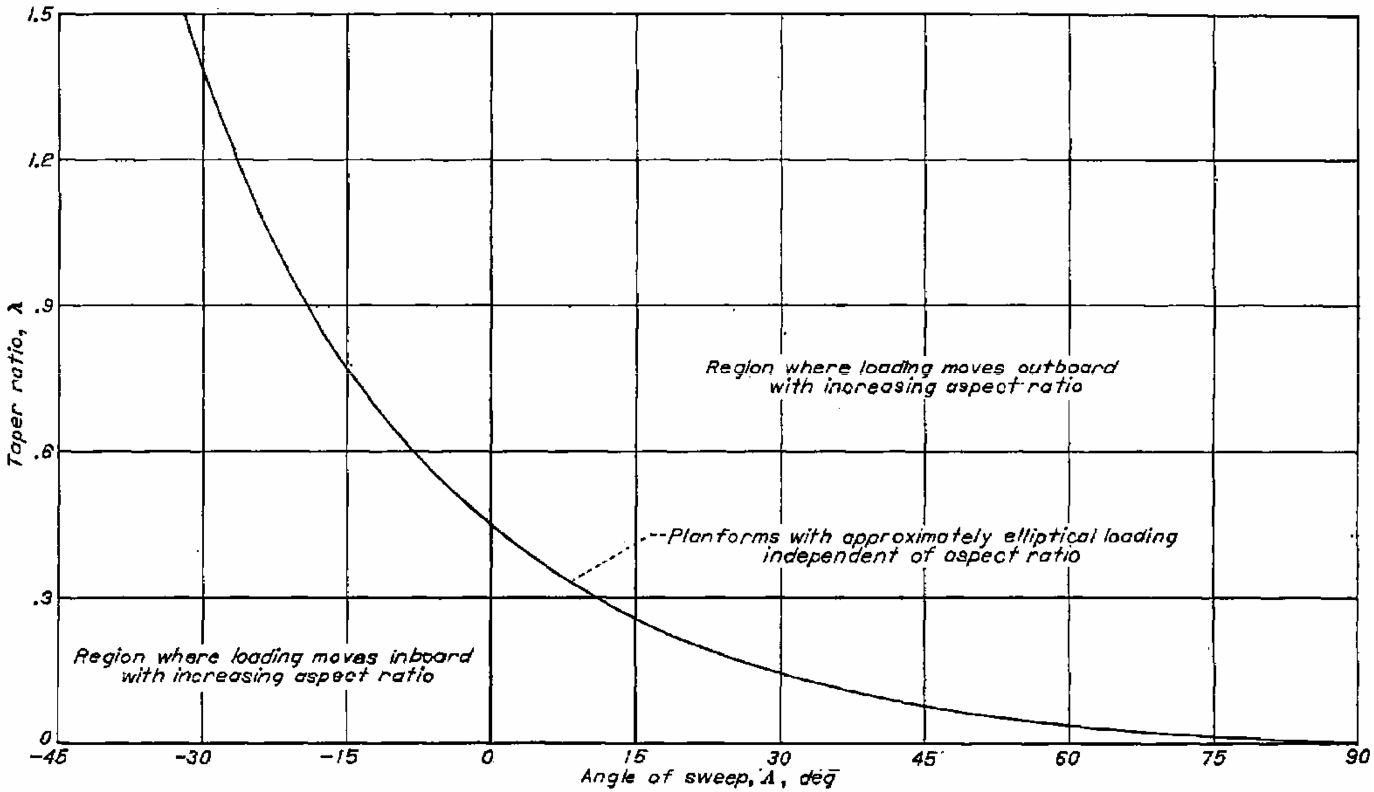


FIGURE 21.—Relation of taper ratio to sweep angle required for approximately elliptical loading.

Method 1: e_{theo}

Optimum Combination of Taper Ratio and Sweep Angle

- For each sweep, there is an optimal taper ratio that minimizes the induced drag.
- NASA Report 921 delivers a curve that relates the taper ratio to sweep for an approximate elliptical loading
- An equation that approximates this curve is $\lambda_{opt} = 0.45 \cdot e^{-0.0375 \cdot \varphi_{25}}$
- The optimum taper ratio for an unswept wing is $\lambda_{opt} = 0.45$
(this is not quite the value from Hörner)

Method 1: e_{theo} for swept Wings

Shifting Hörners curve to a minimum as given by NASA

- The difference in the minimum between Hörner and NASA (sweep angle in degrees):

$$\Delta\lambda = -0.357 + 0.45 \cdot e^{-0.0375 \cdot \varphi_{25}}$$

- Calculating the theoretical Oswald Factor from Hörner's now shifted curve:

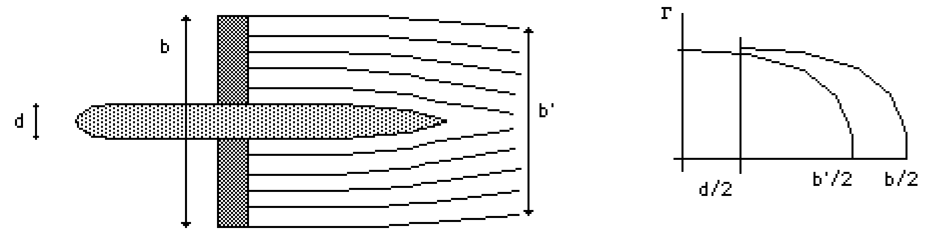
$$e_{theo} = \frac{1}{1 + f(\lambda - \Delta\lambda) \cdot A}$$

Hörner's equation now applied as:

$$f(\lambda - \Delta\lambda) = 0.0524\,(\lambda - \Delta\lambda)^4 - 0.15(\lambda - \Delta\lambda)^3 + 0.1659(\lambda - \Delta\lambda)^2 - 0.0706(\lambda - \Delta\lambda) + 0.0119$$

Literature Study: KROO

Losses due to the Fuselage



- If the flow were axially symmetric and the fuselage were long, then mass conservation leads to

$$b'^2 = b^2 - d_F{}^2$$

- Aspect ratio $A = b^2/S$ is kept constant by convention, so the losses are included into the span efficiency e. The reduction in e is expressed by the factor $k_{e,F}$. So the factor on the span efficiency $k_{e,F}$ is

$$k_{e,F} = \frac{b^2 - d_F{}^2}{b^2} = 1 - \left(\frac{d_F}{b}\right)^2$$

- In practice losses are bigger and experience has shown that induced drag increment is about twice the simple theoretical value, so

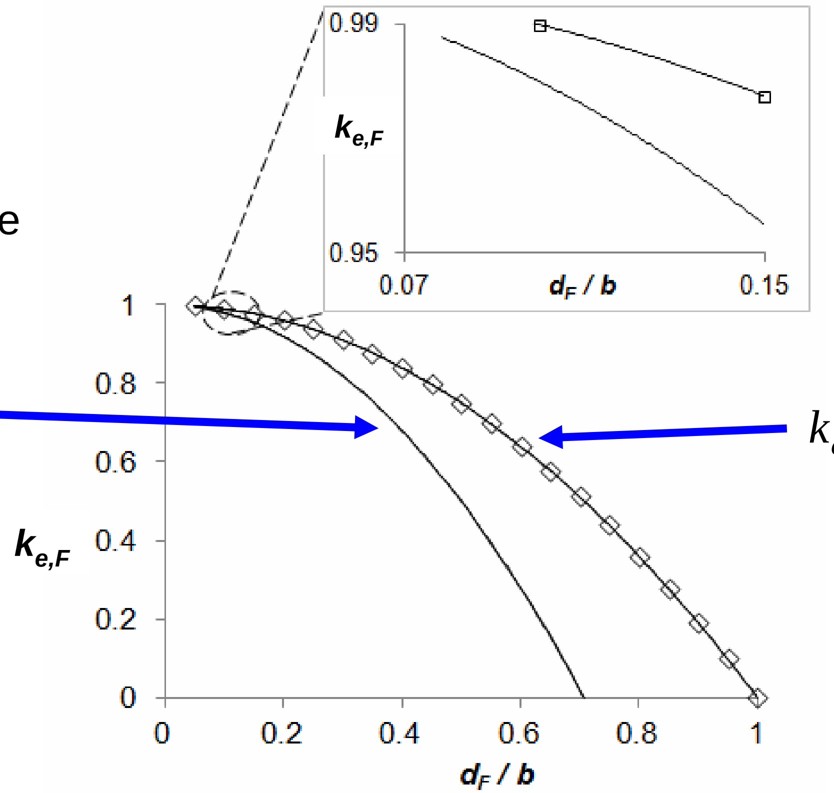
$$k_{e,F} = 1 - 2\left(\frac{d_F}{b}\right)^2$$

Method 1: $k_{e,F}$ Fuselage Correction

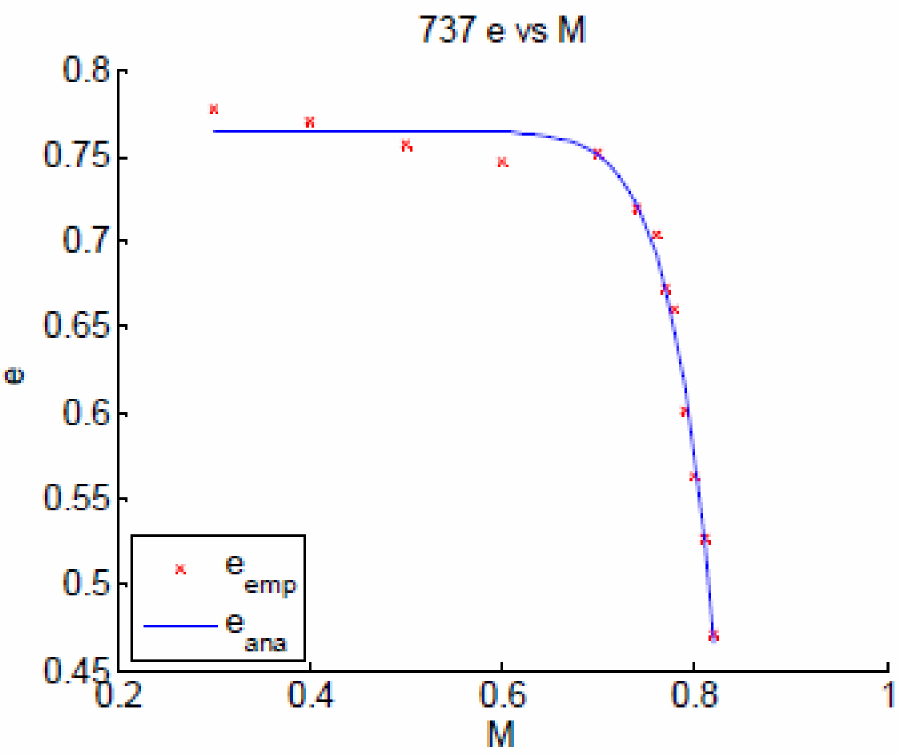
- Fuselage correction factor for the Oswald factor *e* to correct missing or reduced lift due to presents of fuselage

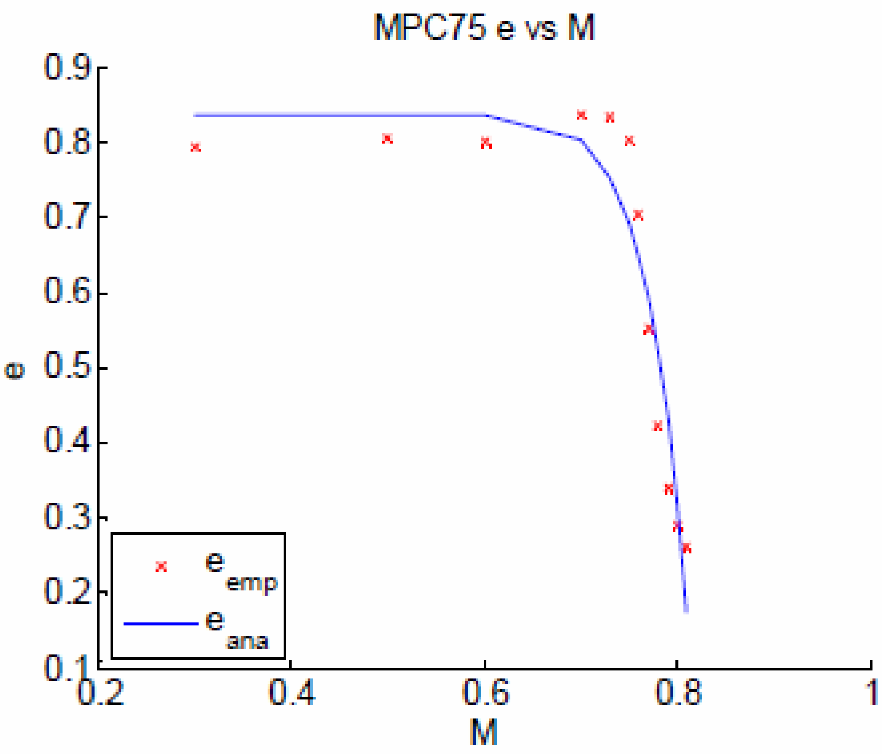
$$k_{e,F} = 1 - 2 \cdot \left(\frac{d_F}{b} \right)^2$$

$$k_{e,F} = 1 - \left(\frac{d_F}{b} \right)^2$$

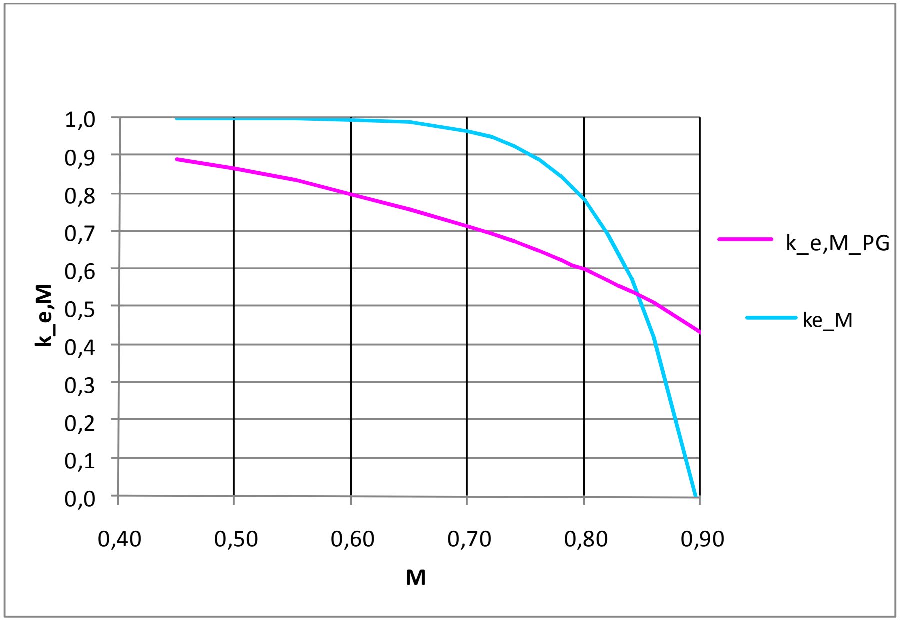


Method 1: Influence of Mach Number: B737 and MPC75





Method 1: Influence of Mach Number - Comparison with Prandtl-Glauert-Correction



$$k_{e,M,PG} = \sqrt{1 - M^2}$$

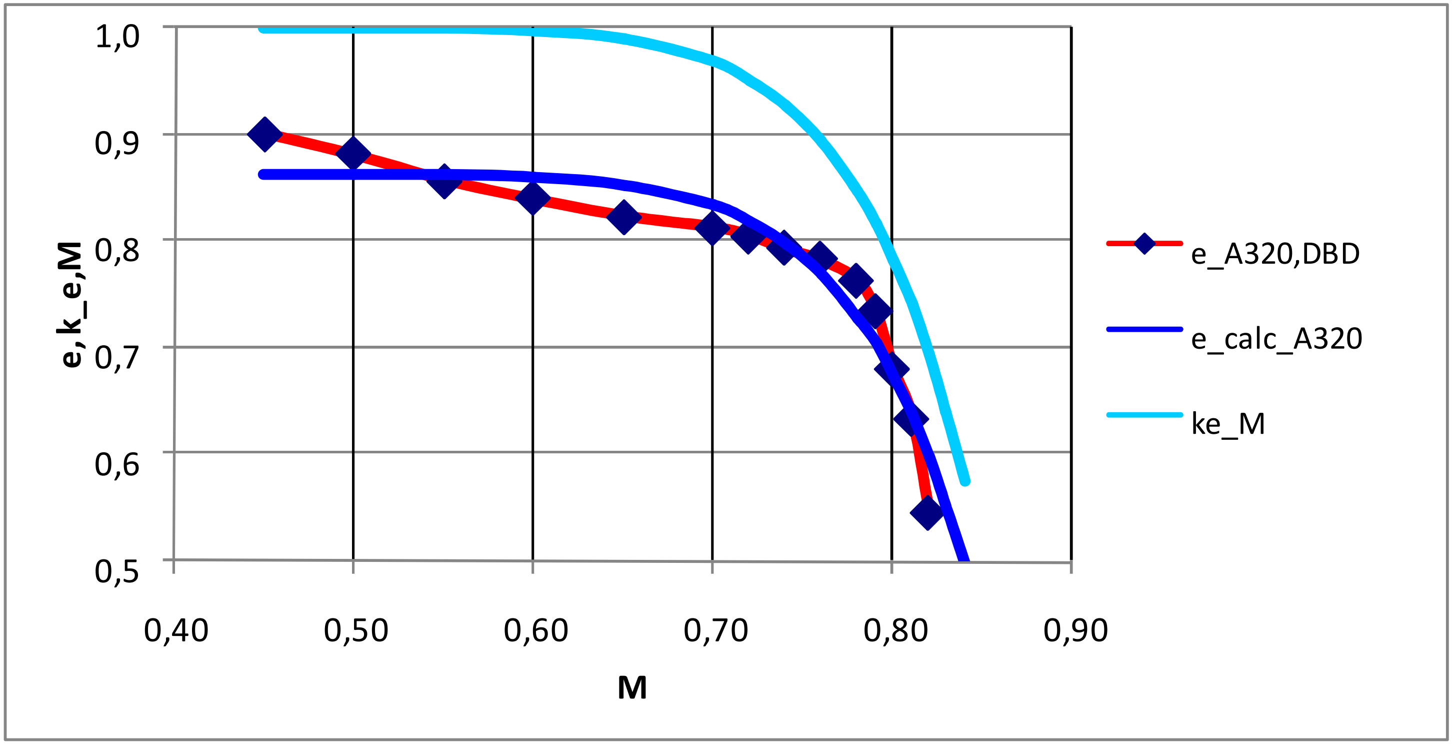
$$k_{e,M} = a_e \left(\frac{M}{M_{comp}} - 1 \right)^{b_e} + c_e$$

$$a_e < 0; \quad c_e = 1$$

- Mach number correction for Oswald factor - from Prandtl-Glauert ($k_{e,M,PG}$) compared with own factor $k_{e,M}$ (using data for the A320)

- $k_{e,M}$ fits the data better than Prandtl-Glauert ($k_{e,M,PG}$) (see next page)

Method 1: Influence of Mach Number and Results for A320



Oswald factor – A320 data and own method. Own Mach number correction $k_{e,M}$

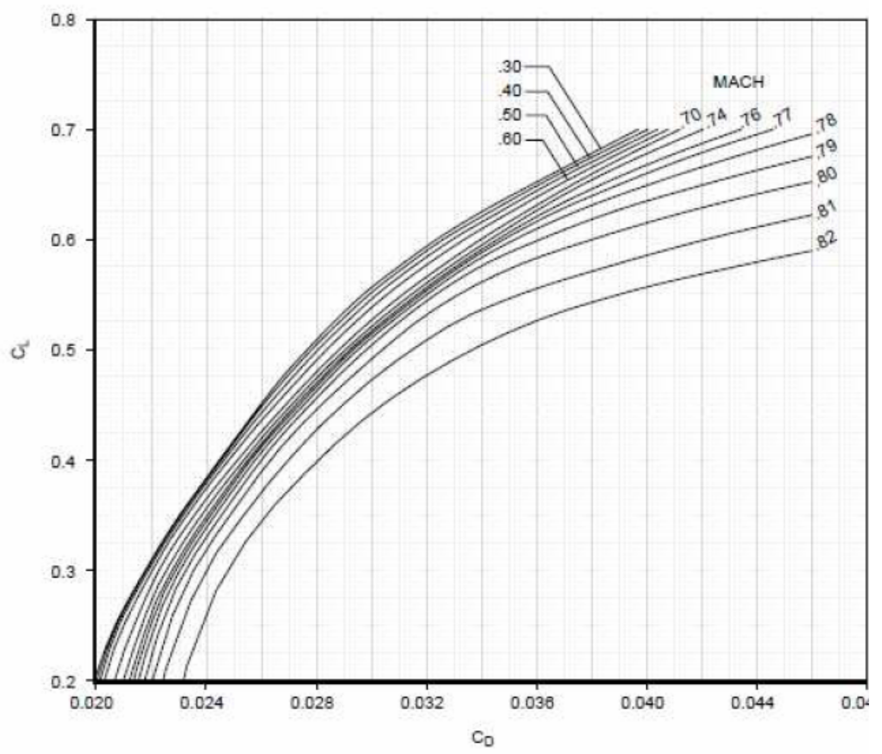
Method 1: Summary: Influence of Parameters

$$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M}$$

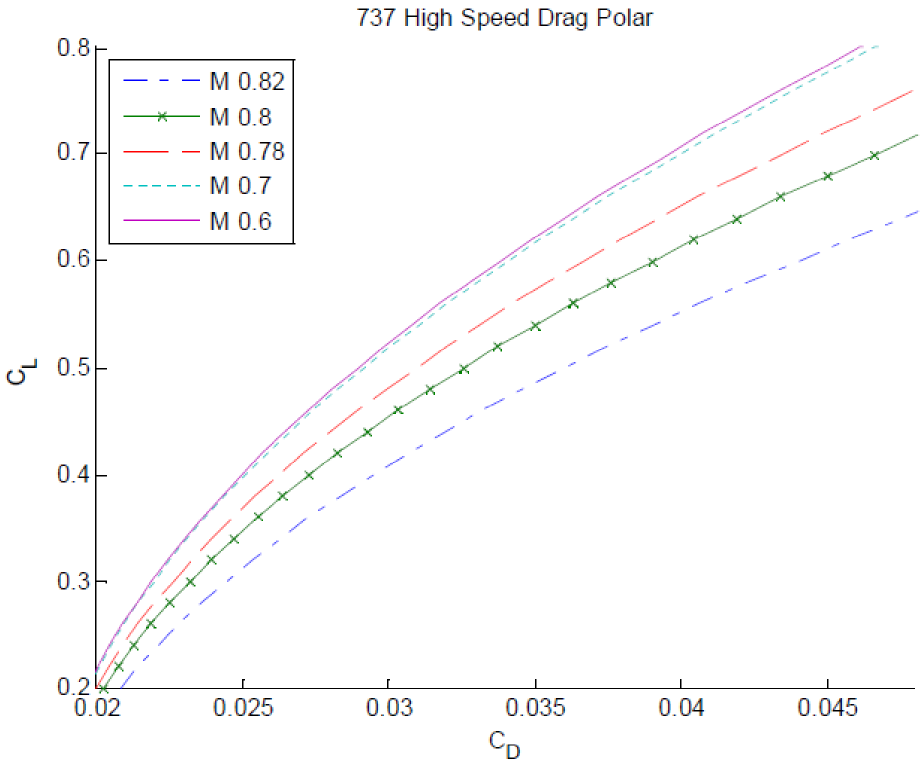
e_{theo}	> 0,9 . Driven by wing parameters: taper, sweep, aspect ratio. Small influence!
$k_{e,F}$	> 0,9 . Driven by fuselage diameter to span
k_{e,D_0}	around 0,85 . Influenced by zero lift drag of the aircraft, but C_{D0} not required
$k_{e,M}$	0,5 … 1,0. Strongest influence on final result of e for cruise of jet aircraft !

Method 1: Results: The Polar of the Boeing 737-800

Aircraft Flight Manual



Calculated



Method 2:

Oswald Factor *e* Calculated with Input of C_{Do} and Twist

Application: Conceptual Design

$$e = \frac{k_{e,M}}{Q + P\pi A} \qquad Q = \frac{1}{e_{theo} \cdot k_{e,F}} \qquad P = K C_{D,0} \qquad K = 0{,}38$$

Method 2: Oswald Factor e Calculated with Input of CDo and Twist

$$e = \frac{k_{e,M}}{Q + P\pi A}$$

$$Q = \frac{1}{e_{theo} \cdot k_{e,F}}$$

$$P = KC_{D,0}$$

K = 0,38

from Method 1

Including the Effect of Twist:

$$P = \frac{C_{L_\alpha} \theta \cdot v}{C_L} + \frac{(C_{L_\alpha} \theta)^2 \cdot w}{C_L{}^2} + KC_{D,0}$$

$$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{A^2 \cdot \left(1 + \tan^2 \varphi_{50} - M^2\right) + 4}}$$

Twist $\theta = i_{W,o} - i_{W,i}$ (is generally negative)

v and w from DUBS

K = 0,38

For A > 4:

$$v = 0.0134(\lambda - 0.3) - 0.0037\lambda^2$$

$$w = \left(0.0088\,\lambda - 0.0051\,\lambda^2\right) \cdot \left(1 - 0.0006A^2\right)$$

v gets negative when λ < 0.33

Theoretical Background

The lift-dependent drag term has two components:

- an **inviscid** part, also called **vortex** drag
- a **viscous** part

$$C_{D,i} = \left(\frac{Q}{\pi A} + P \right) \cdot C_L{}^2$$

$$\frac{1}{\pi A e} = \left(\frac{Q}{\pi A} + P \right)$$

$$\pi A e = \frac{1}{\frac{Q}{\pi A} + P}$$

The term *Q* covers the **inviscid** part of the induced coefficient,

The term *P* is used to express the **viscous** part of induced drag coefficient.

$$e = \frac{1}{Q + P \pi A}$$

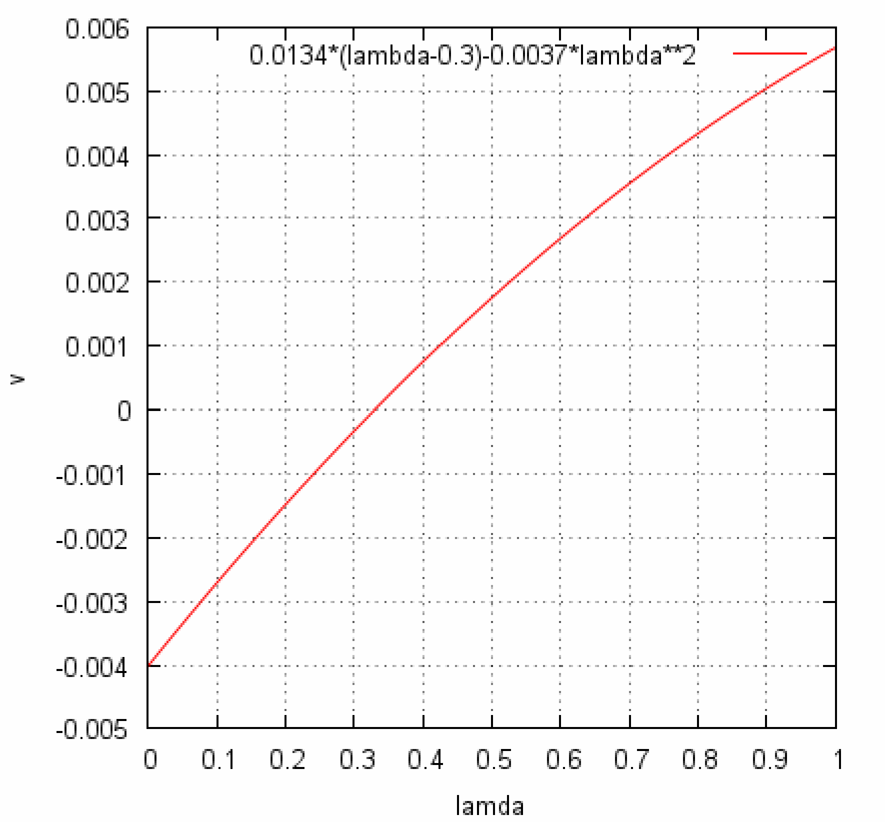
$$e_{inviscid} = 1/Q$$

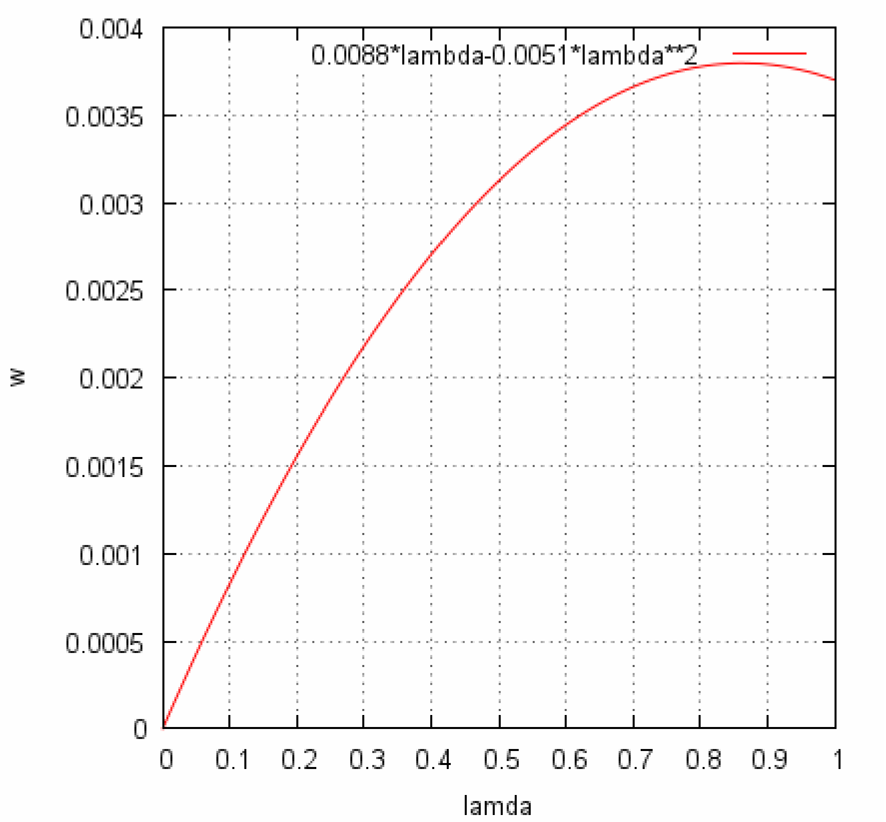
$$C_{D,i} = \frac{Q}{\pi A} C_L{}^2 + K C_{D,0} C_L{}^2 + C_{L_\alpha} \theta \, v \, C_L + (C_{L_\alpha} \theta)^2 w$$

Literature Study: Including the Effects of Twist: v and w from DUBS

$$v = 0.0134(\lambda - 0.3) - 0.0037\lambda^2$$

$$w/(1 - 0.0006A^2) = 0.0088\,\lambda - 0.0051\,\lambda^2$$





Extension of Method 1 and 2:

Oswald Factor *e* for Nonplanar Configurations

$$e_{NP} = e \cdot k_{e,NP}$$

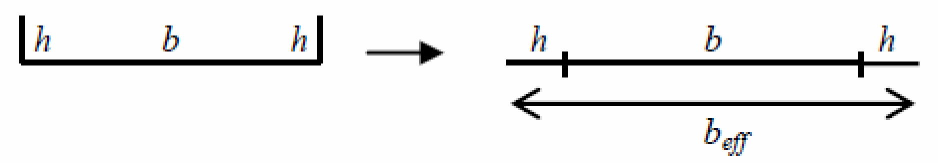
$$e_{NP} = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M} \cdot k_{e,NP}$$

$$e_{NP} = \frac{k_{e,M}}{Q + P\pi A} \cdot k_{e,NP}$$

$$k_{e,NP} = \left(1 + \frac{2}{k_{NP}} \frac{h}{b}\right)^2$$

Estimating the Oswald Factor for Non-Planar Configurations

Wing with winglets



The following relations can be derived from geometry:

$$\frac{b_{eff}}{b} = 1 + 2\frac{h}{b}$$

$$C_{D,i} = \frac{C_L^{\,2}}{\pi A e}$$

$$C_{D,i,WL} = \frac{C_L^{\,2}}{\pi A_{eff}\, e} = \frac{C_L^{\,2}}{\pi A e_{WL}}$$

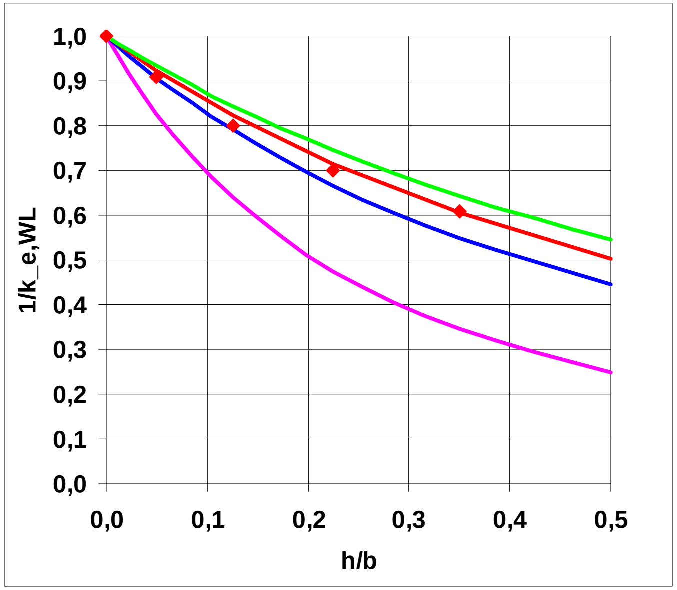
$$e_{WL} = \frac{A_{eff}}{A} \cdot e = \left(\frac{b_{eff}}{b}\right)^2 \cdot e \quad \longrightarrow \quad e_{WL} = \left(1 + 2\frac{h}{b}\right)^2 \cdot e$$

Estimating the Oswald Factor for Non-Planar Configurations

Wing with winglets

Correction of the simple geometrical consideration via the factor k_{WL}

$$e_{WL} = \left(1 + \frac{2}{k_{WL}} \frac{h}{b}\right)^2 \cdot e = k_{e,WL} \cdot e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M} \cdot k_{e,WL}$$



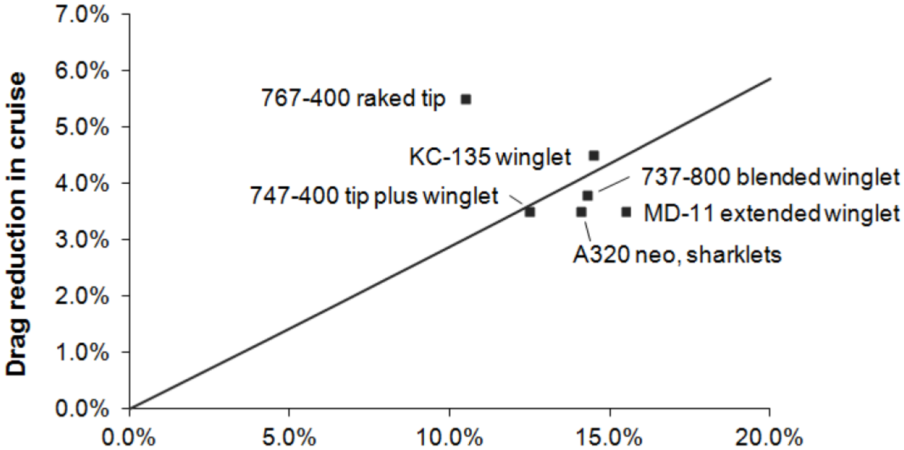
$$k_{e,WL} = \left(1 + \frac{2}{k_{WL}} \frac{h}{b}\right)^2 = \frac{A_{eff}}{A} = \left(\frac{b_{eff}}{b}\right)^2$$

- ◆ DUBS, read from diagram
- geometry, k_wl = 1
- HOWE, k_wl = 2
- DUBS, ZIMMER, k_wl = 2.45
- real A/C average, k_wl = 2.83

Estimating the Oswald Factor for Non-Planar Configurations

Real wings with winglets: Boeing and Airbus data

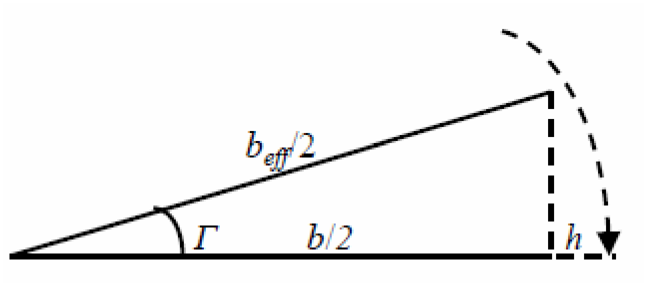
$$e_{WL} = \left(1 + \frac{2}{k_{WL}} \frac{h}{b}\right)^2 \cdot e = k_{e,WL} \cdot e$$



Approach / Source	Reference	k_{WL}
Geometry	-	1.00
Howe	**Howe 2000**	2.00
Kroo	**Kroo 2005**	2.13
Whitcomb	**Whitcomb 1976**	2.20
Dubs, Zimmer	**Dubs 1975, Müller 2003**	2.45
Real aircraft average	**Boeing 2002, Airbus 2012**	2.83
767-400 raked tip	**Boeing 2002**	1.58
747-400 tip plus winglet	**Boeing 2002**	2.92
737-800 blended winglet	**Boeing 2002**	3.08
KC-135 winglet	**Boeing 2002**	2.65
MD-11 extended winglet	**Boeing 2002**	3.62
A320 NEO	**Airbus 2012**	3.29

Estimating the Oswald Factor for Non-Planar Configurations

Wing with dihedral



The following relations can be written:

$$\frac{b}{2} = \frac{b_{eff}}{2} \cdot \cos\Gamma \Rightarrow \frac{b_{eff}}{b} = \frac{1}{\cos\Gamma}$$

$$h = \frac{1}{2}\left(b_{eff} - b\right) \Rightarrow \frac{b_{eff}}{b} = 1 + 2\frac{h}{b}$$

$$\frac{h}{b} = \frac{1}{2}\left(\frac{1}{\cos\Gamma} - 1\right)$$

$$e_{\Gamma} = \frac{A_{eff}}{A} \cdot e$$

$$k_{e,\Gamma} = \left(1 + \frac{2}{k_{WL}} \cdot \frac{h}{b}\right)^2 = \left[1 + \frac{1}{k_{WL}} \cdot \left(\frac{1}{\cos\Gamma} - 1\right)\right]^2$$

$$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M} \cdot k_{e,\Gamma}$$

Estimating the Oswald Factor for Non-Planar Configurations

Wing with dihedral

From the simple geometrical consideration:

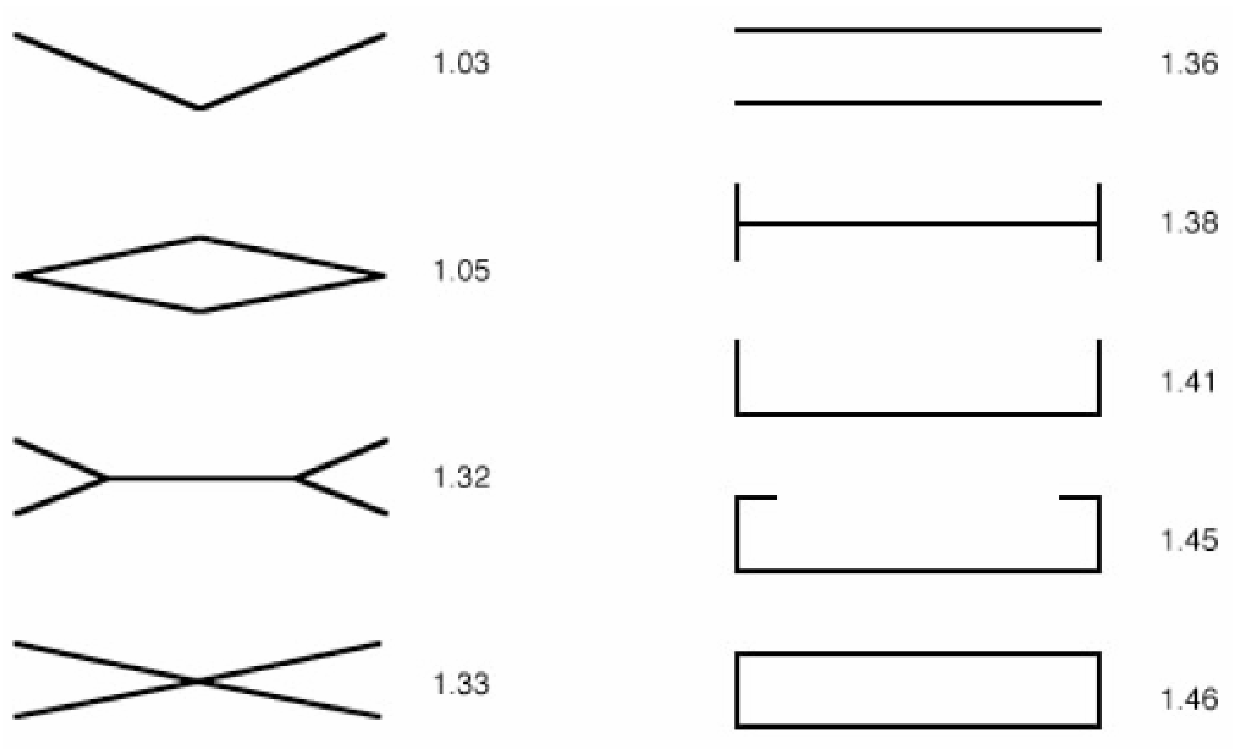
$$k_{e,\Gamma} = \left(1 + 2 \cdot \frac{h}{b}\right)^2 = \left(\frac{1}{\cos\Gamma}\right)^2$$

A more accurate evaluation is achieved by penalizing the relation with the factor k_{WL}

$$k_{e,\Gamma} = \left(1 + \frac{2}{k_{WL}} \cdot \frac{h}{b}\right)^2 = \left[1 + \frac{1}{k_{WL}} \cdot \left(\frac{1}{\cos\Gamma} - 1\right)\right]^2$$

Literature Study: KROO

Various Non-Planar Configurations



Span efficiency for various optimally loaded non-planar systems ($h/b = 0.2$)

KROO

Estimating the Oswald Factor for Non-Planar Configurations

Non-Planar Configurations in General

The following relations can be written: (this time via a penalty factor called k_{NP}

$$e_{NP} = \left(1 + \frac{2}{k_{NP}} \frac{h}{b}\right)^2 \cdot e \Leftrightarrow e_{NP} = k_{e,NP} \cdot e$$

The factor for wings with winglets and dihedral, investigated above, becomes now a particular case of the factor k_{NP}.

Having the $k_{e,NP}$ from KROO, k_{NP} can be calculated for each configuration

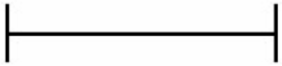
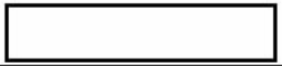
$$k_{NP} = 2 \frac{h}{b} \cdot \frac{1}{\sqrt{k_{e,NP}} - 1}$$

Estimating the Oswald Factor for Non-Planar Configurations

Non-Planar Configurations in General

$$e_{NP} = \left(1 + \frac{2}{k_{NP}} \frac{h}{b}\right)^2 \cdot e = k_{e,NP} \cdot e$$

$$= e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M} \cdot k_{e,NP}$$

	h/b = 0.2	general
Non-planar configuration	$k_{e,NP}$	k_{NP}
	1.03	26.9
	1.05	16.2
	1.32	2.69
	1.33	2.61
	1.36	2.41
	1.38	2.29
	1.41	**2.13**
	1.45	1.96
	1.46	1.92

Estimating the Oswald Factor for Non-Planar Configurations

The Box-Wing Aircraft

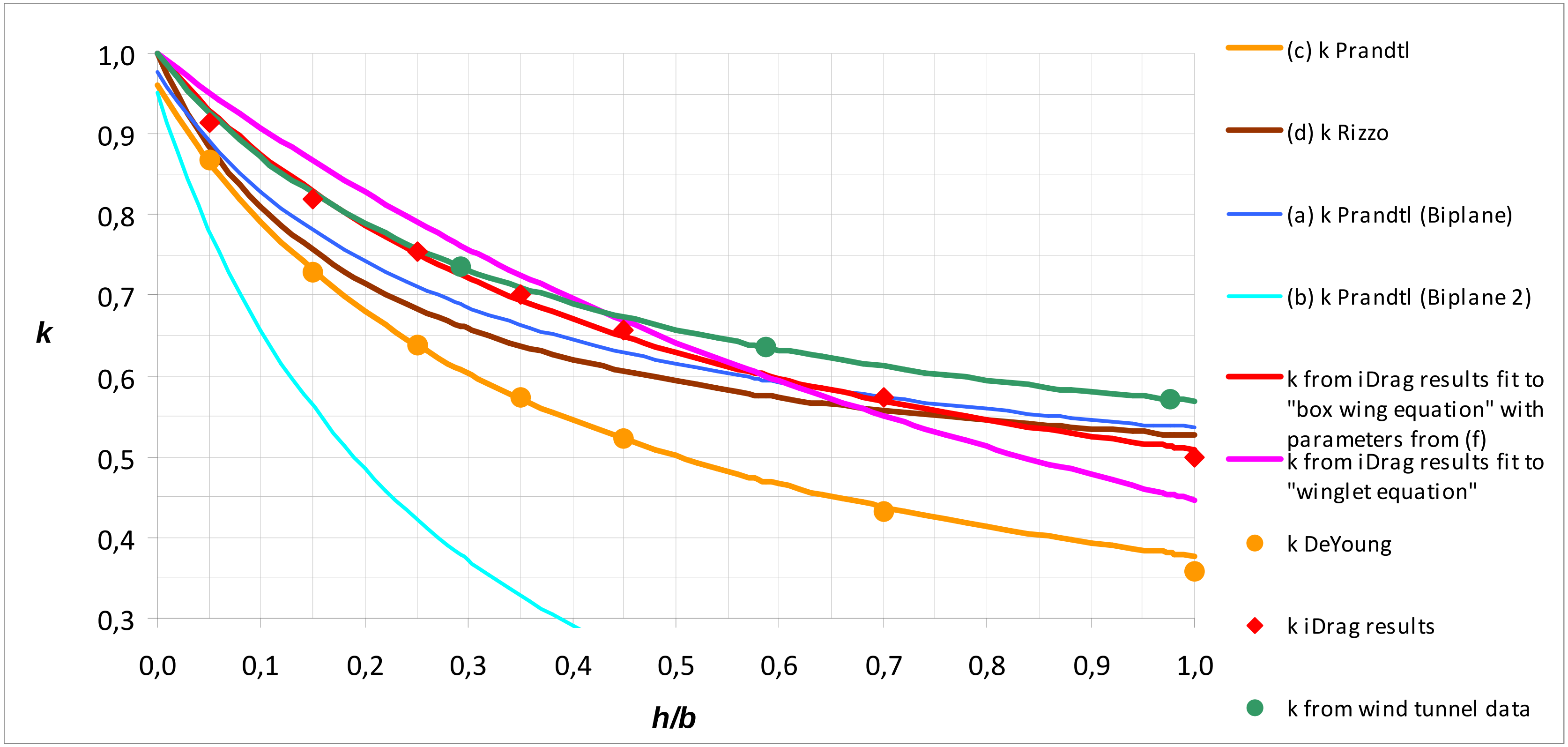
$$\frac{D_{i,box}}{D_{i,ref}} = \frac{e_{ref}}{e_{box}} = k \quad ; \quad \frac{D_{i,box}}{D_{i,ref}} = k = \frac{k_1 + k_2 \cdot h/b}{k_3 + k_4 \cdot h/b} \quad ; \quad \frac{e_{box}}{e_{ref}} = \frac{e_{NP}}{e} = \frac{k_3 + k_4 \cdot h/b}{k_1 + k_2 \cdot h/b}$$

Case	Configuration	Author	k_1	k_2	k_3	k_4	k for $h/b \to 0$	k for $h/b \to \infty$
(a)	Biplane	Prandtl*	1	-0.66	2.1	7.4	0.976	-0.089
(b)	Biplane (2)	Prandtl	1	-0.66	1.05	3.7	0.952	-0.178
(c)	Box wing	Prandtl	1	0.45	1.04	2.81	0.962	0.160
(d)	Box wing	Rizzo	0.44	0.959	0.44	2.22	1	0.432
(e)	Box wing	iDrag best fit	1.304	0.372	1.353	1.988	0.964	0.187
(f)	Box wing	iDrag $k_1 = k_3$	**1.037**	**0.571**	**1.037**	**2.126**	**1**	**0.269**

* here, a different equation is used: $k = 0.5 + \dfrac{k_1 + k_2 \cdot h/b}{k_3 + k_4 \cdot h/b}$.

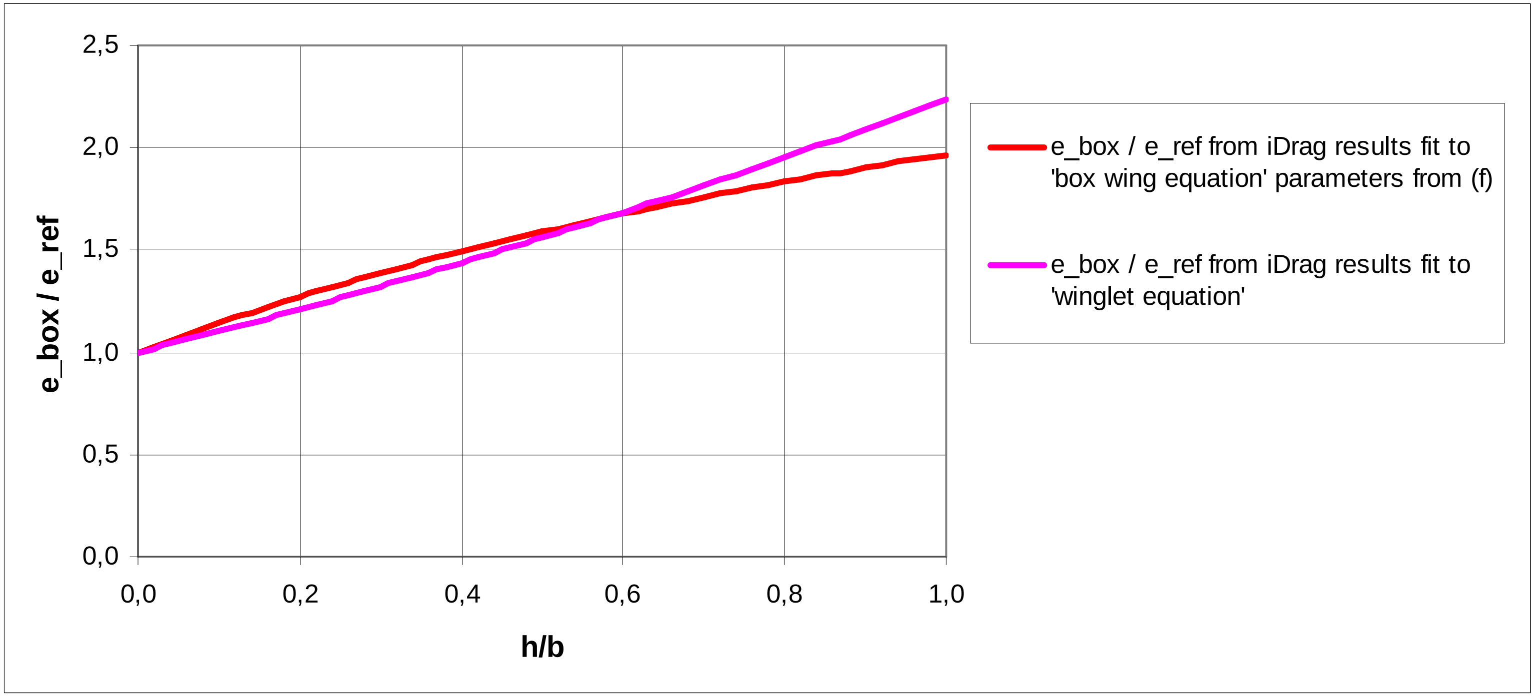
Estimating the Oswald Factor for Non-Planar Configurations

The Box-Wing Aircraft



Estimating the Oswald Factor for Non-Planar Configurations

The Box-Wing Aircraft



Summary and Conclusions

- Simple, physics based method for both conventional and unconventional configurations given
- Treats more in depth the special cases of wings with winglets, wings with dihedral and box wings
- Information provided to assist aircraft designers in making a sufficient accurate estimation of the span efficiency factor *e* during preliminary aircraft design and aircraft design optimization.

ESTIMATING THE OSWALD FACTOR

FROM BASIC AIRCRAFT GEOMETRICAL PARAMETERS

Thank you!

Hochschule für Angewandte
Wissenschaften Hamburg
Hamburg University of Applied Sciences

Contact http://AERO.ProfScholz.de